

- **Advanced Bayesian Econometrics**
- **Homework Assignment #1**
- **Due Date: October 28th 2026**
- **Instructor: Hedibert F. Lopes**
- **Teaching Assistant: Ricardo Pedroso**

Homework 1 is supposed to be done individually and submitted electronically via email to the TA Ricardo Pedroso.

The following exercises are from Chapter 1 and chapter 2 of Reich, Brian, J. and Sujit K. Ghosh. *Bayesian Statistical Methods*. Available from: VitalSource Bookshelf, (2nd Edition). Taylor & Francis, 2026.

CHAPTER 1

16. In a study that uses Bayesian methods to forecast the number of species that will be discovered in future years, [47] report that the number of marine bivalve species discovered each year from 2010-2015 was 64, 13, 33, 18, 30 and 20. Denoting Y_t as the number of species discovered in year t and assuming $Y_t|\lambda \stackrel{iid}{\sim} \text{Poisson}(\lambda)$ and $\lambda \sim \text{Uniform}(0, 100)$, plot the posterior distribution of λ .
17. Assume that (X, Y) follow the bivariate normal distribution and that both X and Y have marginal mean zero and marginal variance one. We observe six independent and identically distributed data points: $(-3.3, -2.6)$, $(0.1, -0.2)$, $(-1.1, -1.5)$, $(2.7, 1.5)$, $(2.0, 1.9)$ and $(-0.4, -0.3)$. Make a scatter plot of the data, and, assuming the correlation parameter ρ has a $\text{Uniform}(-1, 1)$ prior, plot the posterior distribution of ρ .
18. The normalized difference vegetation index (NDVI) is commonly used to classify land cover using remote sensing data. Hypothetically, say that NDVI follows a $\text{Beta}(25, 10)$ distribution for pixels in a rain forest, and a $\text{Beta}(10, 15)$ distribution for pixels in a deforested area now used for agriculture. Assuming about 10% of the rain forest has been deforested, your objective is to build a rule to classify individual pixels as deforested based on their NDVI.
 - (a) Plot the PDF of NDVI for forested and deforested pixels, and the marginal distribution of NDVI averaging over categories.
 - (b) Give an expression for the probability that a pixel is deforested given its NDVI value, and plot this probability by NDVI.
 - (c) You will classify a pixel as deforested if you are at least 90% sure it is deforested. Following this rule, give the range of NDVI that will lead to a pixel being classified as deforested.

Reference [47] is Stewart M. Edie, Peter D. Smits, and David Jablonski. Probabilistic models of species discovery and biodiversity comparisons. *Proceedings of the National Academy of Sciences*, 114(14):3666–3671, 2017.

CHAPTER 2

7. Over the past 50 years California has experienced an average of $\lambda_0 = 75$ large wildfires per year. For the next 10 years you will record the number of large fires in California and then fit a Poisson/gamma model to these data. Let the rate of large fires in this future period, λ , have prior $\lambda \sim \text{Gamma}(a, b)$. Select a and b so that the prior is uninformative with prior variance around 100 and gives prior probability approximately $\text{Prob}(\lambda > \lambda_0) = 0.5$ so that the prior places equal probability on both hypotheses in the test for a change in the rate.
8. An assembly line relies on accurate measurements from an image-recognition algorithm at the first stage of the process. It is known that the algorithm is unbiased, so assume that measurements follow a normal distribution with mean zero, $Y_i | \sigma^2 \stackrel{iid}{\sim} \text{Normal}(0, \sigma^2)$. Some errors are permissible, but if σ exceeds the threshold c , then the algorithm must be replaced. You make $n = 20$ measurements and observe $\sum_{i=1}^n Y_i = -2$ and $\sum_{i=1}^n Y_i^2 = 15$ and conduct a Bayesian analysis with $\text{InvGamma}(a, b)$ prior. Compute the posterior probability that $\sigma > c$ for:
- (a) $c = 1$ and $a = b = 0.1$
 - (b) $c = 1$ and $a = b = 1.0$
 - (c) $c = 2$ and $a = b = 0.1$
 - (d) $c = 2$ and $a = b = 1.0$

For each c , compute the ratio of probabilities for the two priors (i.e. $a = b = 0.1$ and $a = b = 1.0$). Which, if any, of the results are sensitive to the prior?