Fast and Slow Level Shifts in Intraday Stochastic Volatility

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Introduction

Volatility plays a key role in nearly every financial application:

1) Risk Management

   Ex: Markowitz, 1952

2) Volatility trading

   Ex: Stroud and Johannes, 2014

Also useful for macroeconomic policy:

3) Interventions to smooth out exchange rate volatility

   Ex: Fratzscher et al., 2019

What is usually done ?

Traditional intraday SV models have three main characteristics:

1) Persistent volatility

2) Seasonality

3) Announcement effects

Ex: Andersen and Bollerslev, 1998 and Stroud and Johannes, 2014.

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Traditional approach: Inclusion of events based only on prior knowledge.

Probability 1 for events included in the model.

Probability 0 for events excluded from the model.

This paper

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This paper: Data-driven method to determine inclusion probabilities

This paper

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3) Announcement effects (Fast level shifts)

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3) Announcement effects (Fast level shifts)

This paper:

4) Mixed-frequency effects (Slow level shifts)

Volatility evolves over multiple time scales reflecting economic conditions

Ex: Engle and Patton, 2007 and Corsi, 2009.

This paper

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Martins and Lopes (2025)

4) Mixed-frequency effects (Slow level shifts)

This paper - Empirical application

5 minutes returns of West Texas Intermediate (WTI) crude oil.

1) Variance decomposition of h_t

Slow level shifts account between 25% to 39% of the variability.

When events occur, the fast shifts accounts for nearly 25% of the variability.

This paper - Empirical application

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  - Slow level shifts account between 25% to 39% of the variability.
  - When events occur, the fast shifts accounts for nearly 25% of the variability.

2) Events selected by our proposal are

  - Economically plausible
  - Consistent with the idea of ”no news is good news”

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3) Realized volatility forecasts

   Best performance in Mincer–Zarnowitz and Horse-race regressions.

   Improvements are not by chance via Diebold-Mariano test.

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4) Volatility-managed portfolios

   Proposal yields the highest Sharpe ratio.

Related Literature

1) Intraday volatility:

Andersen and Bollerslev, 1997, Andersen and Bollerslev, 1998, Deo, Hurvich, and Lu, 2006, Andersen et al., 2007, Engle, Ghysels, and Sohn, 2013, Stroud and Johannes, 2014, Rossi and Fantazzini, 2015, Martins and Lopes, 2025

2) Volatility over multiple time scales:

Engle and Patton, 2007, Corsi, 2009, Bekierman and Gribisch, 2021,

3) Oil Volatility and its determinants:

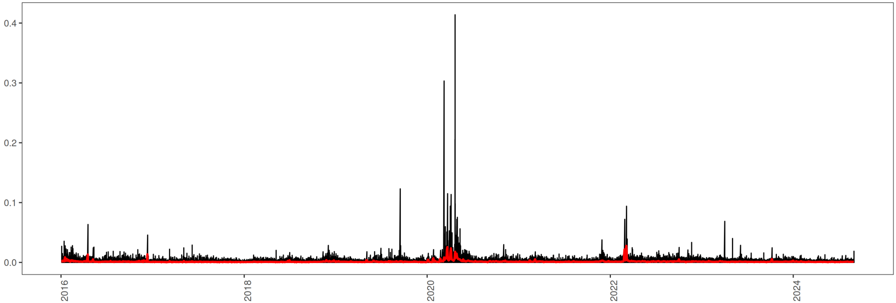
Haugom et al., 2014, Bu, 2014, Žikeš and Baruník, 2015, Bjursell, Gentle, and Wang, 2015, Noguera-Santaella, 2016, Brandt and Gao, 2019, Känzig, 2021, Yang, Zhang, and Chen, 2023

Data

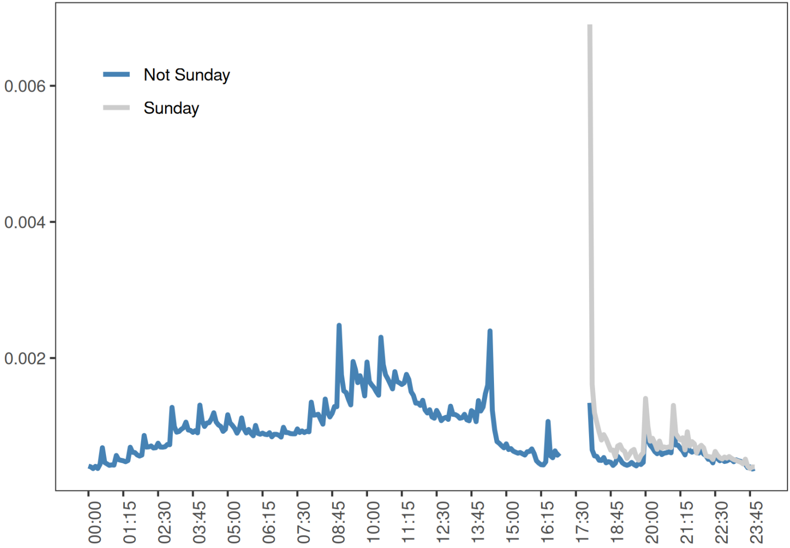
- 5 minutes returns of West Texas Intermediate (WTI) crude oil.
  - Traded 23h hours per day from 18:00 on Sunday until 17:00 on Friday
  - 1-hour maintenance break each day between 17:00 and 18:00.
  - **Sample:** January 3, 2016 to August 30, 2024.
  - **OOS analysis:** January 1, 2022 to August 30, 2024 ($\sim$ 190k observations).
- Announcement data: Bloomberg's Economic Calendar.
  - 94 announcements from US, Germany and China
  - Macroeconomic and commodity related announcements
- Two daily variables
  - CBOE Crude Oil Volatility Index (OVX)
  - CBOE Market Volatility Index (VIX)

EDA - Slow level shift

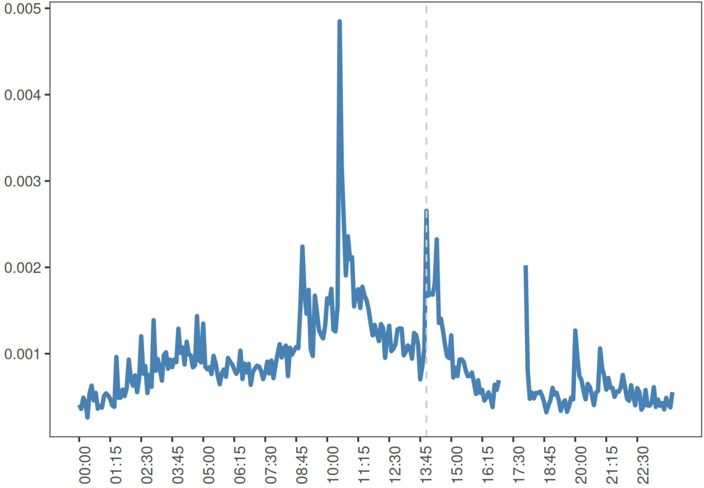
(a) Absolute 5-min Returns and the Locally Weighted Scatterplot Smoothing



EDA - Sunday Open - Events (1/2)

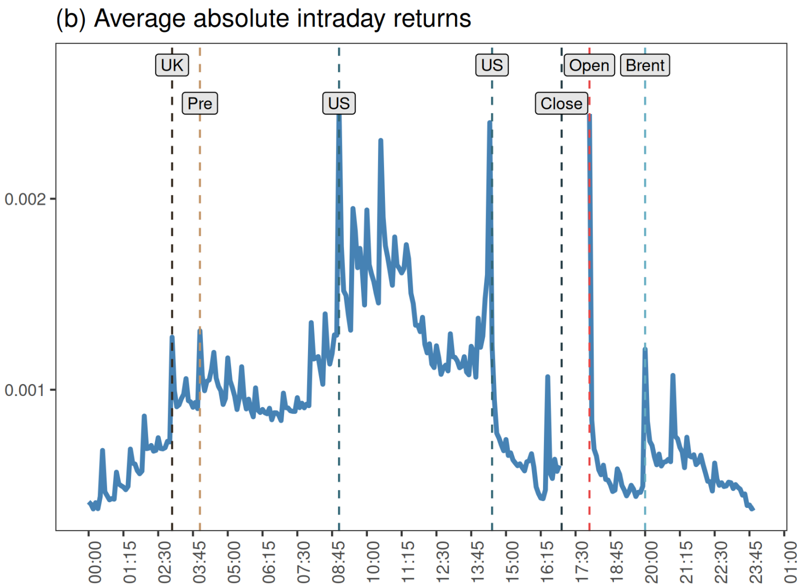


EDA - FOMC - Events (2/2)



FOMC: Federal Open Market Committee

EDA - Intraday seasonality



Proposed model

5-min (log-)returns, y_t, with time varying volatility:

$$y_t = exp(h_t/2)\varepsilon_t \text{ with } \varepsilon_t \sim N(0,1)$$

(Log-)Volatility with 4 components:

$$h_t = (m_\tau + e_t) + p_t + s_t$$

Proposed model - Slow level shift (m_τ)

5-min (log-)returns, y_t, with time varying volatility:

$$y_t = exp(h_t/2)\varepsilon_t \text{ with } \varepsilon_t \sim N(0,1)$$

(Log-)Volatility with 4 components:

$$h_t = (m_\tau + e_t) + p_t + s_t$$

Slow level shift has two components:

$$m_\tau = \underbrace{m_0}_{\text{Long-run level}} + \underbrace{\sum_{j=1}^{J} \delta_j \left[\sum_{l=1}^{L_j} \phi_l(w_j) X_{j,\tau-l} \right]}_{\text{Sensitivy to LF variables}}$$

Restricted beta weighting e.g. Virbickaitė, Nguyen, and Tran, 2023

$$\phi_l(w_j) = \left[1 - \frac{l}{L_j + 1}\right]^{w_j - 1} \left(\sum_{m=1}^{L_j} \left[1 - \frac{m}{L_j + 1}\right]^{w_j - 1} \right)^{-1}$$

Proposed model - Fast level shifts (e_t)

5-min (log-)returns, y_t, with time varying volatility:

$$y_t = exp(h_t/2)\varepsilon_t \text{ with } \varepsilon_t \sim N(0,1)$$

(Log-)Volatility with 4 components:

$$h_t = (m_\tau + e_t) + p_t + s_t$$

$$e_t = \sum_{i=1}^{N} I_{it}\alpha_i$$

Events inclusion are driven by a spike-and-slab structure:

$$\alpha|\pi \sim (1-\pi)\delta_0 + \pi N(0, \sigma_\alpha^2)$$

$$\pi \sim Bern(\gamma)$$

Examples of I_t:

t	Ann. 1	Ann. 2	Ann. 3	...	Ann. A
2022 - Aug - 10 00:00 - 00:05	1	0	0	...	0
2022 - Aug - 10 00:05 - 00:10	0	1	0	...	0
2022 - Aug - 10 00:10 - 00:15	1	0	1	...	0

Proposed model - Remaining components (p_t and s_t)

Returns, y_t, with time varying volatility:

$$y_t = exp(h_t/2)\varepsilon_t \text{ with } \varepsilon_t \sim N(0,1)$$

(Log-)Volatility with 4 components:

$$h_t = (m_\tau + e_t) + p_t + s_t$$

Persistent volatility:

$$p_t = \beta^p p_{t-1} + \sigma_\eta \eta_t$$

Intraday seasonal component:

$$s_t = \sum_{k=1}^{K} H_{tk}\beta_k \text{ with } \sum_{k=1}^{K} \beta_k = 0$$

Proposed model

$$y_t = exp(h_t/2)\varepsilon_t \text{ with } \varepsilon_t \sim N(0,1)$$

$$h_t = (m_\tau + e_t) + p_t + s_t$$

$$\left.\begin{aligned} & m_\tau = m_0 + \sum_{j=1}^{J} \delta_j \left[\sum_{l=1}^{L_j} \phi_l(w_j) X_{j,\tau-l}\right] \\ & \phi_l(w_j) = \left[1 - \frac{l}{L_j+1}\right]^{w_j-1} \left(\sum_{m=1}^{L_j} \left[1 - \frac{m}{L_j+1}\right]^{w_j-1}\right)^{-1} \end{aligned}\right\} \text{Slow level shifts}$$

$$\left.\begin{aligned} & e_t = \sum_{i=1}^{N} I_{it}\alpha_i \\ & \alpha|\pi \sim (1-\pi)\delta_0 + \pi N(0,\sigma_\alpha^2) \\ & \pi \sim Bern(\gamma) \end{aligned}\right\} \text{Fast level shifts}$$

$$p_t = \beta^p p_{t-1} + \sigma_\eta \eta_t$$

$$s_t = \sum_{k=1}^{K} H_{tk}\beta_k \text{ with } \sum_{k=1}^{K} \beta_k = 0$$

Model typology

	Persistence (p_t)	Seasonal (s_t)	Fast shift (e_t)	Slow shift (m_τ)
SSVA MIDAS	✓	✓	✓	✓
SSVA	✓	✓	✓	
SSVAg	✓	✓	✓	
SSV	✓	✓		
SV	✓			

Priors and MCMC

Let $\Theta = (\beta^p, \sigma_\eta^2, \{p_t\}_{t=1}^T, \{\beta_k\}_{k=1}^{K-1}, \alpha, \pi, \sigma_a^2, \gamma, m_0, \{w_j\}_{j=1}^J, \delta)$

$$p(\Theta|y) \propto p(y|\Theta)p(\Theta)$$

We break down $p(\Theta|y)$ into:

- $\beta^p|\cdot$: Conjugated normal
- $\sigma_\eta^2|\cdot$: Conjugated IG
- $\{p_t\}_{t=1}^T|\cdot$: Kim, Shephard, and Chib, 1998 mixture of Gaussians approach
- $\{\beta_k\}_1^{K-1}|\cdot$: Conjugated normal
- $(\alpha, \pi)|\cdot$: Geweke, 1996 to avoid problems due to the Dirac's delta.
- $\sigma_a^2|\cdot$: Conjugated IG
- $\gamma|\cdot$: Conjugated beta
- $m_0|\cdot$: Conjugated normal
- $\{w_j\}_{j=1}^J|\cdot$: Metropolis-Hastings
- $\delta|\cdot$: Conjugated normal

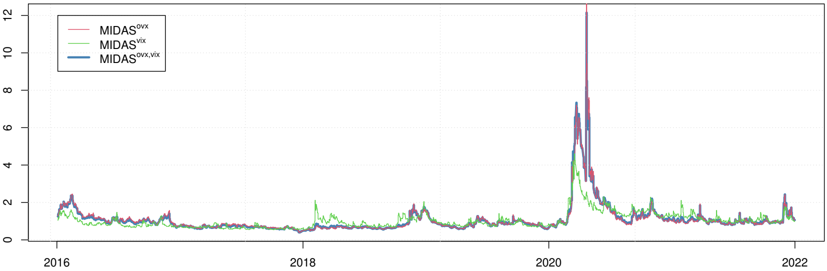
Variance decomposition

Slow level shifts accounts for a large share of the variation in h_t.

	m_τ	e_t	p_t	s_t
SSVA MIDAS^{ovx,vix}	39.07	0.68	20.28	39.98
SSVA MIDAS^{ovx}	38.62	0.69	20.71	39.98
SSVA MIDAS^{vix}	25.60	0.68	34.14	39.58

The table reports the variance decomposition of the log-variance, h_t, at the posterior modes into its four components: slow (m_τ) and fast (e_t) level shifts, latent persistence (p_t) and intraday seasonality (s_t). Entries are percentage shares.

Slow level shift: $\exp(m_\tau/2)$



Conditional variance decomposition

When an event occurs, fast level shifts accounts for a large share of the variation in h_t.

	m_τ	e_t	p_t	s_t
SSVA MIDAS^{ovx,vix}	30.19	22.47	17.09	30.24
SSVA MIDAS^{ovx}	29.71	22.65	17.33	30.31
SSVA MIDAS^{vix}	19.27	22.75	27.19	30.79

This table reports the variance decomposition of the log-variance, h_t, into its four components when at least one event occurs: slow (m_τ) and fast (e_t) level shifts, latent persistence (p_t) and intraday seasonality (s_t). The model parameters are fixed in their posterior modes. Entries are percentage shares.

Naive vs spike and slab event selection

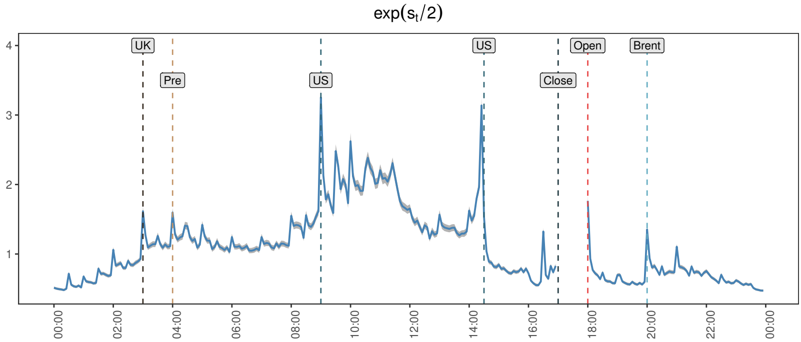
Naive models lead to strange events being included e.g US condition cotton

SS selection is consistent with ”no news is good news”

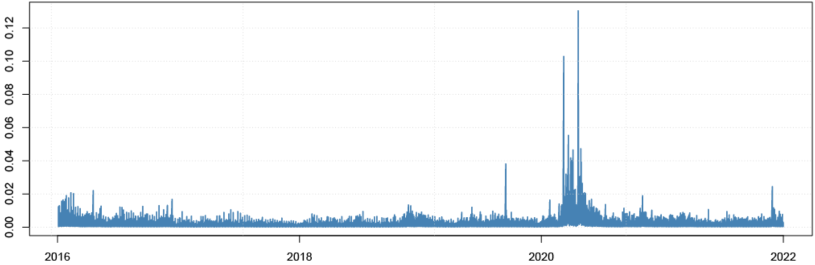
	OLS estimate	OLS p-value	Post mode	Post. inclusion
GE GfK Consumer Confidence	-1.69	0.01	0.00	0.04
GE HCOB Germany Services PMI	1.13	0.03	0.00	0.02
GE ZEW Survey Current Situation	-1.31	0.04	0.00	0.18
US Change in Nonfarm Payrolls	2.01	0.00	1.11	1.00
US Condition Cotton	-1.08	0.09	0.00	0.12
US CPI MoM	0.92	0.17	-0.00	0.90
US DOE Crude Oil Inventories	2.83	0.00	2.27	1.00
US FOMC Rate Decision	1.53	0.05	1.27	1.00
US ISM Manufacturing	1.23	0.06	0.00	0.16
US U. of Michigan Sentiment	-0.17	0.72	0.00	0.84
Sunday Open	2.75	0.00	2.09	1.00

The table presents the OLS announcement coefficients and the corresponding p-values of regression $\log(y_t^2) \sim \log(y_{t-1}^2) + s_t + e_t$, the MCMC posterior announcement coefficients (mode) using the SSVA MIDAS^{ovx,vix} model and posterior inclusion probabilities.

Intraday seasonality: $\exp(\beta/2)$



Evolution of the volatility



Model parameters

Accounting for MIDAS component diminishes the SV persistence

Consistent with the HAR of Corsi, 2009 but now in a HF environment

	m_0	β^p	σ_η	γ	σ_α	δ_{ovx}	δ_{vix}
SSVA MIDAS^{ovx,vix}	-14.199	0.788	0.296	0.112	1.080	0.672	0.129
SSVA MIDAS^{ovx}	-14.202	0.798	0.283	0.116	1.072	0.763	–
SSVA MIDAS^{vix}	-14.153	0.928	0.128	0.123	1.052	–	0.648
SSVA	-14.192	0.973	0.072	0.132	1.067	–	–

The table presents the posterior MCMC medians for the parameters of the models considered.

Realized volatility forecasts

We compare the out-of-sample performance of our proposal in multiple ways

1) Mincer-Zarnowitz regression

$$RVol_i = \alpha_0 + \alpha_1 \widehat{vol}_{i|i-1}^{model} + \varepsilon_i,$$

2) Horse-race regression

$$RVol_i = \beta_0 + \beta_1 \widehat{vol}_{i|i-1}^{benchmark} + (1 - \beta_1)\widehat{vol}_{i|i-1}^{competitor} + \varepsilon_i,$$

3) Diebold-Mariano test under squared and absolute losses

Realized volatility forecasts

	Mincer–Zarnowitz			Horse-Race		Diebold-Mariano	
Model	$\hat{\alpha}_0$	$\hat{\alpha}_1$	R^2	$\hat{\beta}_1$	$t(\hat{\beta}_1)$	Squared	Absolute
Panel A: 5-min-ahead forecasts							
SSVA MIDAS^{ovx}	0.009	0.939	0.473	–	–	–	–
SSVA MIDAS^{ovx,vix}	0.009	0.931	0.472	1.117	24.645	-2.85***	-15.5***
SSVA MIDAS^{vix}	0.012	0.881	0.471	0.695	63.520	-4.72***	-20.77***
SSVA	0.013	0.889	0.471	0.594	71.937	-3.60***	-12.67***
SSVAg	0.013	0.882	0.470	0.614	75.858	-4.31***	-14.98***
SSV	0.013	0.884	0.455	0.698	104.285	-2.86***	-16.15***
SV	0.013	0.896	0.389	0.810	182.932	-6.85***	-53.89***
GARCH	0.018	0.779	0.356	0.758	243.396	-10.75***	-69.64***
AR1-RV	0.017	0.849	0.304	0.798	263.429	-7.45***	-74.37***
Panel B: Day-ahead forecasts							
SSVA MIDAS^{ovx}	-0.001	1.087	0.593	–	–	–	–
SSVA MIDAS^{ovx,vix}	0.000	1.032	0.585	0.801	2.979	-0.24	-1.40*
SSVA MIDAS^{vix}	0.003	0.811	0.371	1.063	19.997	-7.28***	-11.99***
SSVA	-0.091	4.930	0.205	1.082	29.506	-6.75***	-14.97***
SSVAg	-0.086	4.589	0.180	1.083	29.647	-7.13***	-15.96***
SSV	-0.214	10.079	0.348	1.080	29.767	-6.68***	-15.44***
SV	-0.139	7.095	0.302	1.078	29.371	-6.32***	-14.36***
GARCH	-0.027	2.886	0.436	1.016	23.811	-8.35***	-14.30***
AR1-RV	-0.221	12.855	0.065	1.082	31.332	-6.65***	-13.54***

Volatility-managed portfolios

Proposed by Moreira and Muir (2017)

Improved performance by adjusting exposure based on forecasted volatility

$$y^*_{t+1} = \frac{c}{\widehat{vol}_{t+1|t}} y_{t+1},$$

Take leveraged positions when volatility is low.

Out of the market when volatility is high.

For WTI is easy (and plausible) to be leveraged.

Volatility-managed portfolios

Volatility management improves Sharpe Ratio

Our proposal improves the most

	Mean	*SR*
WTI	0.053	0.132
SSVA MIDAS^{ovx}	0.213	0.531
SSVA MIDAS^{ovx,vix}	0.211	0.526
SSVA MIDAS^{vix}	0.214	0.532
SSVA	0.194	0.484
SSVAg	0.189	0.469
SSV	0.204	0.508
SV	0.097	0.242
GARCH	0.075	0.188
AR1-RV	0.101	0.252

1-step-ahead volatility-managed portfolio. Mean is annualized average return, *SR* is the annualized Sharpe ratio.

Conclusion

We propose a mixed-frequency stochastic volatility model

1) Slow level shifts from daily variables

2) Fast level shifts from events

Level shifts explain large part of the variability of h_t

Forecasting improvements for both 5-minutes and day-ahead horizons

Volatility-managed portfolio from our forecasts yield the highest Sharpe ratio

Muito obrigado!

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