Choosing trousers and ties:

A Lovely Introduction to Bayesian Statistics

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Source[1]

These slides follow Keith McNulty,
"A Lovely Introduction to Bayesian Statistics,"
Medium, 2 May 2026.

`https://blog.keithmcnulty.org/a-lovely-introduction-to-bayesian-statistics-809ac7619152`

The problem at the heart of the article is taken from the *Sixth Term Examination Paper (STEP) Further Mathematics A, 1990, Question 16*. This is a Cambridge University entrance examination.

[1] These slides were prepared with the help of Claude-for-Mac.

Lesson zero: when in doubt, write it down

> *"If you don't immediately know what you are doing, take the time to write down the information you have. Often in the course of doing so, a method will reveal itself."*
>
> — Keith McNulty

We'll see this happen *twice* in the next few slides.

The problem (informally)

Someone has rules for choosing trousers and ties:

- **Brown trousers:** equally likely to wear any tie, *except* the Cravat.
- **Grey trousers:** same rule as Brown.
- **Jeans on a weekday:** half the time, no tie; the other half, picks a tie uniformly at random (Cravat included).
- **Jeans on a Sunday:** always wears a tie, picked uniformly at random.

Photographic evidence: the subject is wearing a black tie.

Question. What is the probability that the photo was taken on a Sunday?

Step 1: write the information into a table

Rows: trouser choice.

Columns: tie choice.

Each cell: $P(\text{tie} \mid \text{trousers})$.

Trousers	**Black**	Tie 2	Tie 3	Cravat	No tie
Brown	1/3	1/3	1/3	0	0
Grey	1/3	1/3	1/3	0	0
Jeans (weekday)	1/8	1/8	1/8	1/8	1/2
Jeans (Sunday)	1/4	1/4	1/4	1/4	0

Each row sums to 1.

The only column we need is **Black**.

Background assumption: The three trouser types are equally likely on any given day, and trouser choice is independent of the day of the week.

Step 2: spot the "given that"

"Any use of the term given that invites the use of Bayes' Theorem."

We are after $P(\text{Sunday} \mid \text{Black tie})$.

Bayes' theorem. For any two events A and B,

$$P(A \mid B) = \frac{P(B \mid A)\,P(A)}{P(B)}.$$

With $A = \text{Sunday}$ and $B = \text{Black tie}$,

$$P(\text{Sunday} \mid \text{Black tie}) = \frac{P(\text{Black tie} \mid \text{Sunday})\,P(\text{Sunday})}{P(\text{Black tie})}.$$

Classical vs Bayesian, in one paragraph

- **Classical statistics** asks "What is the probability of this event?" *without* an explicit prior context.
- **Bayesian statistics** asks "What is the probability of this event, *given the context I already know*?"

The context is the prior.
The updated probability after seeing data is the posterior.

Bayes' theorem is the rule that turns "prior + likelihood" into the "posterior".

Three quantities to compute

$$P(\text{Sunday} \mid \text{Black tie}) = \frac{\overbrace{P(\text{Black tie} \mid \text{Sunday})}^{\text{likelihood}} \cdot \overbrace{P(\text{Sunday})}^{\text{prior}}}{\underbrace{P(\text{Black tie})}_{\text{marginal}}}.$$

We compute them in turn:

1. $P(\text{Black tie} \mid \text{Sunday})$
2. $P(\text{Sunday})$
3. $P(\text{Black tie})$

(1) $P(\text{Black tie} \mid \text{Sunday})$

On Sunday the three trouser types are equally likely ($\frac{1}{3}$ each):

$$\begin{aligned}
P(\text{Black tie} \mid \text{Sunday}) &= P(\text{Black tie, Brown trouser} \mid \text{Sunday}) \\
&+ P(\text{Black tie, Grey trouser} \mid \text{Sunday}) \\
&+ P(\text{Black tie, Jeans} \mid \text{Sunday}) \\
&= P(\text{Brown trouser} \mid \text{Sunday})P(\text{Black tie} \mid \text{Brown trouser, Sunday}) \\
&+ P(\text{Grey trouser} \mid \text{Sunday})P(\text{Black tie} \mid \text{Grey trouser, Sunday}) \\
&+ P(\text{Jeans} \mid \text{Sunday})P(\text{Black tie} \mid \text{Jeans, Sunday}) \\
&= \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{4} = \frac{11}{36} \approx 30.6\%
\end{aligned}$$

(2) P(Sunday) and (3) P(Black)

Sunday is one day in seven:

$$P(\text{Sunday}) = \frac{1}{7} \approx 14.3\%$$

Black tie: sum over all (trouser, day) combinations:

$$P(\text{Black}) = \underbrace{\tfrac{1}{3}\cdot\tfrac{1}{3}}_{\text{Brown}} + \underbrace{\tfrac{1}{3}\cdot\tfrac{1}{3}}_{\text{Grey}} + \underbrace{\tfrac{6}{7}\cdot\tfrac{1}{3}\cdot\tfrac{1}{8}}_{\text{Jeans, weekday}} + \underbrace{\tfrac{1}{7}\cdot\tfrac{1}{3}\cdot\tfrac{1}{4}}_{\text{Jeans, Sunday}}$$

$$= \frac{2}{9} + \frac{1}{28} + \frac{1}{84} = \frac{17}{63} \approx 26.98\%$$

The posterior

$$
\begin{aligned}
P(\text{Sunday} \mid \text{Black}) &= \frac{P(\text{Black} \mid \text{Sunday})\, P(\text{Sunday})}{P(\text{Black})} \\
&= \frac{\frac{11}{36} \cdot \frac{1}{7}}{\frac{17}{63}} = \frac{11}{68} \approx 16.2\%.
\end{aligned}
$$

Reading:

Without the photo, you'd say $P(\text{Sunday}) = 1/7 \approx 14.3\%$.

After seeing the black tie, the posterior nudges up to about 16.2%.

The evidence is mildly informative.

Updating the priors

> *"In real life, the information we have access to changes all the time. If our priors change, this will usually have an impact on the probability of the event we are interested in."*

New information. There is now *one* weekday (call it Monday) on which our subject is *certain* to wear jeans, with the usual weekday tie habits.

Which quantities change?

- $P(\text{Black} \mid \text{Sunday})$ — no, Sundays are unaffected.
- $P(\text{Sunday})$ — no, still $1/7$.
- $P(\text{Black})$ — yes, because one weekday is now "always jeans".

Updated information

- **Monday** (probability 1/7): *always* jeans, weekday tie habits.
- **Tuesday-Saturday** (probability 5/7): trousers uniform on $\{\text{Brown}, \text{Grey}, \text{Jeans}\}$; jeans behave on weekday.
- **Sunday** (probability 1/7): trousers uniform on $\{\text{Brown}, \text{Grey}, \text{Jeans}\}$; jeans behave on Sunday.

Brown and Grey are now possible on 6/7 of days; jeans are possible *every* day.

New $P(\text{Black})$

Summing over (day, trouser):

$$P(\text{Black}) = \underbrace{\frac{6}{7}\cdot\frac{1}{3}\cdot\frac{1}{3}}_{\text{Brown}} + \underbrace{\frac{6}{7}\cdot\frac{1}{3}\cdot\frac{1}{3}}_{\text{Grey}} + \underbrace{1\cdot\frac{1}{7}\cdot\frac{1}{8}}_{\text{Mon, jeans}}$$

$$+ \underbrace{\frac{1}{3}\cdot\frac{5}{7}\cdot\frac{1}{8}}_{\text{Tue-Sat, jeans}} + \underbrace{\frac{1}{7}\cdot\frac{1}{3}\cdot\frac{1}{4}}_{\text{Sun, jeans}}$$

$$= \frac{4}{21} + \frac{8}{168} + \frac{1}{84} = \frac{21}{84} = \boxed{\frac{1}{4}}.$$

The updated posterior

Plug the new $P(\text{Black}) = \frac{1}{4}$ into Bayes:

$$P(\text{Sunday} \mid \text{Black}) = \frac{11}{36} \cdot \frac{1}{7} \cdot 4 = \boxed{\frac{11}{63} \approx 17.5\%}.$$

Side-by-side.

	Original	**Updated**
$P(\text{Sunday})$ *(prior)*	$1/7 \approx 14.3\%$	$1/7 \approx 14.3\%$
$P(\text{Sunday} \mid \text{Black})$ *(posterior)*	$11/68 \approx 16.2\%$	$11/63 \approx 17.5\%$

Same prior on Sunday, same likelihood of black-on-Sunday — but a richer model of the rest of the week shifts the posterior.

Why Bayesian statistics is important

- Classical methods often assume a *uniform* prior — every context is equally plausible. Computationally simple, but throws away information.
- Bayesian methods condition on the prior context you actually have, and update it as new evidence arrives.
- Historically expensive; the modern computational revolution (MCMC, variational methods, GPUs) has made it routine.

> *"This is how our brains work in general — we adapt to new information all the time — so Bayesian methods are a real step forward in modeling our comprehension of the world."*
>
> — Keith McNulty

Takeaways

1. "Given that" is a flag for Bayes' theorem.
2. Identify your prior, your likelihood, and your marginal — the rest is arithmetic.
3. When the priors change, the posterior changes. That feature, not a bug, is what makes Bayesian inference powerful.

Recommended next step:
Richard McElreath, *Statistical Rethinking* (2nd ed., CRC Press).

Thank you.