

Blackboard from my today's class (April 16th 2026) about normal dynamic linear models (NDLM), and Dynamic generalized linear models (DGLM)

NDLM

$\{y_1, \dots, y_T\}$ ← Dados

$\{F_1, \dots, F_T\}$

$y_t = F_t^T \beta_t + v_t$ $v_t \stackrel{iid}{\sim} N(0, V)$

$\beta_t = G_t \beta_{t-1} + w_t$ $w_t \sim N(0, W)$

Geralmente $G_t = I_p$

LLM: $y_t = \beta_t + v_t$ $p=1, F_t=G_t=1, \forall t$
 $\beta_t = \beta_{t-1} + w_t$

AR(1)+noise $p=1, F_t=1, G_t=\phi, \forall t$
 $y_t = \beta_t + v_t$
 $\beta_t = \alpha + \phi \beta_{t-1} + w_t$

Linear Growth $F_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, G_t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \forall t$
 $y_t = \beta_t + v_t$
 $\beta_t = \begin{pmatrix} \beta_{t-1} + \alpha \\ \beta_{t-1} \end{pmatrix} + w_t$
 $\beta_t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \beta_{t-1} + \begin{pmatrix} \alpha \\ 0 \end{pmatrix} + w_t$

Ex2: $y_t | \pi_t \sim \text{Ber}(\pi_t)$
 $E(y_t | \pi_t) = \pi_t$ Dynamic Logistic Regression
 $\pi_t = F_t^T \beta_t$
 $\log\left(\frac{\pi_t}{1-\pi_t}\right) = F_t^T \beta_t \Leftrightarrow \pi_t = \frac{1}{1+e^{-F_t^T \beta_t}}$
 $\beta_t = G_t \beta_{t-1} + w_t$ $w_t \sim N(0, W)$

Ex2: $p(y_t | \pi_t) = \pi_t^{y_t} (1-\pi_t)^{1-y_t}$
 $= (1+e^{-F_t^T \beta_t})^{-y_t} \left(1 - \frac{1}{1+e^{-F_t^T \beta_t}}\right)^{1-y_t}$
 $= (1+e^{-F_t^T \beta_t})^{-y_t} \left(\frac{e^{-F_t^T \beta_t}}{1+e^{-F_t^T \beta_t}}\right)^{1-y_t}$
 $= \frac{(1+e^{-F_t^T \beta_t})^{-y_t} e^{-F_t^T \beta_t (1-y_t)}}{(1+e^{-F_t^T \beta_t})^{-y_t}}$

$y | \lambda \sim \text{Poi}(\lambda)$
 $p(y|\lambda) = \frac{\lambda^y e^{-\lambda}}{y!}$ $y=0,1,2,\dots$
 $p(y|\lambda) = \left(\frac{1}{y!}\right) \exp(-y \log \lambda - \lambda)$
 $a(y) \exp(y \theta - \psi(\theta))$

Timeline of methods:

- 1960/1970: NLDM
- 1970/1971: MH
- 1970/1980: Metodo MC (1954-1980)
- 1985/1995: DGLM (West, Harrison, Migon, 85)
- 1988/1995: Gibbs/Data Augmentation, Metropolis-Hastings, CMCMC
- 1995-2000: MCMC / DGLM

Normalidade \rightarrow Família exponencial
 Gamma, Beta, Binomial, Poisson, Normal, etc

$y_t = F_t^T \beta_t + v_t \Leftrightarrow y_t | F_t, \beta_t, v_t \sim N(F_t^T \beta_t, V)$

$E(y_t | F_t, \beta_t) = F_t^T \beta_t$

Ex1: $y_1, \dots, y_T \sim \text{Poisson}(\lambda_t)$ $\lambda_t \geq 0$
Ex2: $y_1, \dots, y_T \sim \text{Bernoulli}(\pi_t)$ $\pi_t \in (0,1)$

Ex1: $y_t | \lambda_t \sim \text{Poi}(\lambda_t)$
 $\log(\lambda_t) = F_t^T \beta_t \Leftrightarrow \lambda_t = e^{F_t^T \beta_t}$
 $\beta_t = G_t \beta_{t-1} + w_t$ $w_t \sim N(0, W)$

Nonlinear DM

$y_t = \frac{x_t^2}{20} + v_t$ $v_t \sim N(0, \sigma^2)$
 $x_t = \alpha x_{t-1} + \beta \left(\frac{x_{t-1}}{1+x_{t-1}^2}\right) + \gamma \cos(1.2t) + w_t$ $w_t \sim N(0, \tau^2)$

$\Theta = (\sigma^2, \tau^2, \alpha, \beta, \gamma) \in (\mathbb{R}^+)^2 \times (\mathbb{R})^3$ $p(\sigma^2) p(\alpha, \beta, \gamma | \sigma^2) p(\tau^2)$
 $\mathcal{X} = (x_1, \dots, x_T)$ $x_0 \sim N(m_0, C_0)$