$\boxed{\text{First order NDLM \& FFBS}}$

The posterior for the parameter $\theta \in \mathbb{R}^n$ in a 1st order DLM is obtained by at least two alternative ways. The model is

$$y_t|\theta_t \sim N(\theta_t, \sigma^2) \quad \text{conditionally independent } t=1,2,\dots,n$$

$$t=2,\dots,n \quad \theta_t|\theta_{t-1} \sim N(\theta_{t-1},\tau^2) \qquad \theta_1|D_0 \sim N(0,\tau^2)$$

$\boxed{\text{APPROACH 1}}$

$$y=\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad \theta=\begin{pmatrix} \theta_1 \\ \vdots \\ \theta_n \end{pmatrix} \qquad y|\theta \sim N(0, \sigma^2 I_n) \qquad \theta \sim N(0, \tau^2 \Omega_n)$$

where
$$(\Omega_n)_{ij} = \begin{cases} i & j \geq i \\ j & j < i \end{cases} \Rightarrow (\Omega_n^{-1})_{ij} = \begin{cases} 2 & i=j \\ -1 & |i-j|=1 \\ 0 & \text{otherwise} \end{cases}$$

$$\Rightarrow p(\theta|y) \propto \exp\left\{-\frac{1}{2} \frac{\theta^T \Omega_n^{-1} \theta}{\tau^2}\right\} \exp\left\{-\frac{1}{2}\left(\theta^T\theta/\sigma^2 - 2\theta^T y/\sigma^2\right)\right\}$$

$$\Rightarrow (\theta|y) \sim N\left[\left(\Omega_n^{-1}/\tau^2 + I_n/\sigma^2\right)^{-1} y \,;\, \left(\Omega_n^{-1}/\tau^2 + I_n/\sigma^2\right)^{-1}\right]$$

$\boxed{\text{APPROACH 2}}$ Forward filtering backward sampling

For $\theta_0|D_0 \sim N(m_0, C_0) \Rightarrow t=1,\dots,n$ $(\theta_{t-1}|D_{t-1}) \sim N(m_{t-1}, C_{t-1})$

$$(\theta_t|D_t) \sim N(m_t, C_t)$$

$$m_t = (1-A_t)m_{t-1} + A_t y_t$$

$$C_t = A_t \sigma^2 \qquad A_t = \frac{(C_{t-1}+\tau^2)}{(C_{t-1}+\tau^2+\sigma^2)}$$

For $t = n-1, n-2, \dots, 3, 2, 1$

$$\theta_t|D_n \sim N(m_t^n, C_t^n)$$

$$m_t^n = (1-B_t)m_t + B_t m_{t+1}^n$$

$$C_t^n = (1-B_t)C_t + B_t^2 C_{t+1}^n \qquad \text{with } m_n^n = m_n, \; C_n^n = C_n$$