

ADVANCED
BAYESIAN ECONOMETRICS QUIZ

24/09/2024

$$y_1, \dots, y_m | \theta \text{ iid Poi}(\theta) \quad \text{i.e. } p(y_i) = \frac{\theta^{y_i} e^{-\theta}}{y_i!}$$

$$\theta \sim \text{Gamma}(\alpha, \beta) \quad \alpha, \beta \text{ known} \quad \theta > 0 \quad y_i = 0, 1, 2, \dots$$

a) Derive $p(y_1, \dots, y_m)$ — prior predictive

b) Derive $p(\theta | y_1, \dots, y_m)$ — posterior density

c) Derive $p(y_{m+1} | y_1, \dots, y_m)$

$$\text{Recall: } \theta \sim G(\alpha, \beta) \Rightarrow p(\theta) = \frac{\beta^\alpha}{\Gamma(\alpha)} \theta^{\alpha-1} e^{-\beta\theta}, \quad \theta, \alpha, \beta > 0.$$