First homework assignment

PhD in Business Economics — Advanced Bayesian Econometrics

Hedibert Freitas Lopes — Due date: 9am, September 3rd, 2024.

Please submit either your file (handwritten or typed) in PDF or HTML. The file must be a single PDF/HTML document for submission to me at `hedibertfl@insper.edu.br`. Students should follow the deadlines for submissions. This homework assignment should be done individually.

iid Bernoulli model with a mixture of Betas prior

Let us continue with the iid Bernoulli model worked in our first class, along with the dataset

```
n = 10
y = c(0,0,0,1,0,1,0,1,0,1)
s = sum(y==1)
```

Now assume that the prior for θ is a 2-component mixture of Betas, i.e.

$$\theta|\gamma \sim \pi Beta(a,b) + (1-\pi)Beta(c,d),$$

for known hyperparameters $\gamma = (\pi, a, b, c, d)$.

i) Show that $p(\theta|s,n,\gamma)$ is also a 2-component mixture of Betas:

$$\theta|s,n,\gamma \sim \pi_1 Beta(a_1,b_1) + (1-\pi_1)Beta(c_1,d_1),$$

ii) Derive π_1, a_1, b_1, c_1 and d_1.

iii) Assume that $a = 2$ and $b = 5$, as in the original Beta prior worked in class. Also, let $c = 5$ and $d = 2$, as well as $\pi = 0.5$. Plot in the same frame, both prior densities (beta prior and mixture of betas prior) and both posterior densities (also beta posterior and mixture of betas posterior). Critically comment the results.