Second homework assignment

Professional Master in Economics — Bayesian Learning

Hedibert Freitas Lopes — Due date: 7:30pm, May 21st, 2024.

Please submit either your handwritten or typed file in PDF or HTML. In case of typing, it is suggested to use Rmarkdown. The file must be a single PDF/HTML document for submission to the TA's email at `luizatv@insper.edu.br`. Students should follow the deadlines for submissions. This homework assignment should be done individually.

SIR for Bayesian inference

For some likelihood $p(\texttt{data}|\theta)$ and prior $p(\theta)$, we have the following posterior for $\theta \in (0,1)$:

$$p(\theta|\texttt{data}) \propto (1+\theta)^{125}(1-\theta)^{38}\theta^{34}.$$

Use sampling importance resampling (SIR) to obtain Monte Carlo-based approximations to the following posterior quantities:

1. $E(\theta|\texttt{data})$
2. $\sqrt{V(\theta|\texttt{data})}$
3. $Pr(\theta < 0.6|\texttt{data})$
4. $q_{0.05}$, where $Pr(\theta < q_{0.05}|\texttt{data}) = 0.05$.
5. $q_{0.95}$, where $Pr(\theta < q_{0.95}|\texttt{data}) = 0.95$.

As $\theta \in (0,1)$, the simplest proposal distribution would be the $q_1(\theta) \equiv U(0,1)$. However, if you draw $p(\theta)$ you will notice that virtually all posterior density of θ lies in the interval $(0.5, 0.9)$, so a "*better*" proposal would be $q_2(\theta) \equiv U(0.5, 0.9)$. Compare both approximations when computing the above 5 summaries. In order to make your life easier, let us assume first that i) $M = 10,000$ and $N = 10,000$. Repeat everything with iii) $M = 100,000$ and $N = 10,000$, and then with iii) $M = 1,000,000$ and $N = 10,000$. Here M is the sample size from the proposal and N is the resample size from the M proposal draws, so N is always less than or equal to M.

	$q_1 \equiv U(0,1)$			$q_2 \equiv U(0.5,0.9)$		
	$M = 10^4$	$M = 10^5$	$M = 10^6$	$M = 10^4$	$M = 10^5$	$M = 10^6$
$E(\theta\|\texttt{data})$						
$\sqrt{V(\theta\|\texttt{data})}$						
$Pr(\theta < 0.6\|\texttt{data})$						
$q_{0.05}$						
$q_{0.95}$						