Stochastic Volatility Models

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Stochastic volatility model

The canonical stochastic volatility model (SV-AR(1), hereafter), is

$$
\begin{aligned}
y_t &= e^{h_t/2}\varepsilon_t \\
h_t &= \mu + \phi h_{t-1} + \tau\eta_t
\end{aligned}
$$

where ε_t and η_t are $N(0,1)$ shocks with $E(\varepsilon_t\eta_{t+h}) = 0$ for all h and $E(\varepsilon_t\varepsilon_{t+l}) = E(\eta_t\eta_{t+l}) = 0$ for all $l \neq 0$.

τ^2: volatility of the log-volatility.

$|\phi| < 1$ then h_t is a stationary process.

Let $y^n = (y_1, \ldots, y_n)'$, $h^n = (h_1, \ldots, h_n)'$ and $h_{a:b} = (h_a, \ldots, h_b)'$.

Prior information

Uncertainty about the initial log volatility is $h_0 \sim N(m_0, C_0)$.

Let $\theta = (\mu, \phi)'$, then the prior distribution of (θ, τ^2) is normal-inverse gamma, i.e. $(\theta, \tau^2) \sim NIG(\theta_0, V_0, \nu_0, s_0^2)$:

$$\begin{aligned} \theta|\tau^2 &\sim N(\theta_0, \tau^2 V_0) \\ \tau^2 &\sim IG(\nu_0/2, \nu_0 s_0^2/2) \end{aligned}$$

For example, if $\nu_0 = 10$ and $s_0^2 = 0.018$ then

$$\begin{aligned} E(\tau^2) &= \frac{\nu_0 s_0^2/2}{\nu_0/2 - 1} = 0.0225 \\ Var(\tau^2) &= \frac{(\nu_0 s_0^2/2)^2}{(\nu_0/2-1)^2(\nu_0/2-2)} = (0.013)^2 \end{aligned}$$

Hyperparameters: m_0, C_0, θ_0, V_0, ν_0 and s_0^2.

Posterior inference

The SV-AR(1) is a dynamic model and posterior inference via MCMC for the the latent log-volatility states h_t can be performed in at least two ways.

Let $h_{-t} = (h_{0:(t-1)}, h_{(t+1):n})$, for $t = 1, \ldots, n-1$ and $h_{-n} = h_{1:(n-1)}$.

- Individual moves for h_t
  - $(\theta, \tau^2 | h^n, y^n)$
  - $(h_t | h_{-t}, \theta, \tau^2, y^n)$, for $t = 1, \ldots, n$
- Block move for h^n
  - $(\theta, \tau^2 | h^n, y^n)$
  - $(h^n | \theta, \tau^2, y^n)$

Sampling $(\theta, \tau^2 | h^n, y^n)$

Conditional on $h_{0:n}$, the posterior distribution of (θ, τ^2) is also normal-inverse gamma:

$$(\theta, \tau^2 | y^n, h_{0:n}) \sim NIG(\theta_1, V_1, \nu_1, s_1^2)$$

where $X = (1_n, h_{0:(n-1)})$, $\nu_1 = \nu_0 + n$

$$\begin{aligned} V_1^{-1} &= V_0^{-1} + X'X \\ V_1^{-1}\theta_1 &= V_0^{-1}\theta_0 + X'h_{1:n} \\ \nu_1 s_1^2 &= \nu_0 s_0^2 + (y - X\theta_1)'(y - X\theta_1) + (\theta_1 - \theta_0)'V_0^{-1}(\theta_1 - \theta_0) \end{aligned}$$

Sampling $(h_0|\theta, \tau^2, h_1)$

Combining

$$h_0 \sim N(m_0, C_0)$$

and

$$h_1|h_0 \sim N(\mu + \phi h_0, \tau^2)$$

leads to (by Bayes' theorem)

$$h_0|h_1 \sim N(m_1, C_1)$$

where

$$\begin{aligned} C_1^{-1} m_1 &= C_0^{-1} m_0 + \phi \tau^{-2}(h_1 - \mu) \\ C_1^{-1} &= C_0^{-1} + \phi^2 \tau^{-2} \end{aligned}$$

Conditional prior distribution of h_t

Given h_{t-1}, θ and τ^2, it can be shown that, for $t = 1, \ldots, n-1$,

$$\begin{pmatrix} h_t \\ h_{t+1} \end{pmatrix} \sim N\left\{ \begin{pmatrix} \mu + \phi h_{t-1} \\ (1+\phi)\mu + \phi^2 h_{t-1} \end{pmatrix}, \tau^2 \begin{pmatrix} 1 & \phi \\ \phi & (1+\phi^2) \end{pmatrix} \right\}$$

so $E(h_t | h_{t-1}, h_{t+1}, \theta, \tau^2)$ and $V(h_t | h_{t-1}, h_{t+1}, \theta, \tau^2)$ are

$$\begin{aligned} \mu_t &= \left(\frac{1-\phi}{1+\phi^2}\right)\mu + \left(\frac{\phi}{1+\phi^2}\right)(h_{t-1} + h_{t+1}) \\ \nu^2 &= \tau^2(1+\phi^2)^{-1} \end{aligned}$$

respectively. Therefore,

$$\begin{aligned} (h_t | h_{t-1}, h_{t+1}, \theta, \tau^2) &\sim N(\mu_t, \nu^2) \qquad t = 1, \ldots, n-1 \\ (h_n | h_{n-1}, \theta, \tau^2) &\sim N(\mu_n, \tau^2) \end{aligned}$$

where $\mu_n = \mu + \phi h_{n-1}$.

Sampling h_t via random walk Metropolis

Let $\nu_t^2 = \nu^2$ for $t = 1, \ldots, n-1$ and $\nu_n^2 = \tau^2$, then

$$p(h_t | h_{-t}, y^n, \theta, \tau^2) \quad = \quad f_N(h_t; \mu_t, \nu_t^2) f_N(y_t; 0, e^{h_t})$$

for $t = 1, \ldots, n$.

A simple random walk Metropolis algorithm with tuning variance v_h^2 would work as follows:

For $t = 1, \ldots, n$

1. Current state: $h_t^{(j)}$
2. Sample h_t^* from $N(h_t^{(j)}, v_h^2)$
3. Compute the acceptance probability

$$\alpha = \min\left\{1, \frac{f_N(h_t^*; \mu_t, \nu_t^2) f_N(y_t; 0, e^{h_t^*})}{f_N(h_t^{(j)}; \mu_t, \nu_t^2) f_N(y_t; 0, e^{h_t^{(j)}})}\right\}$$

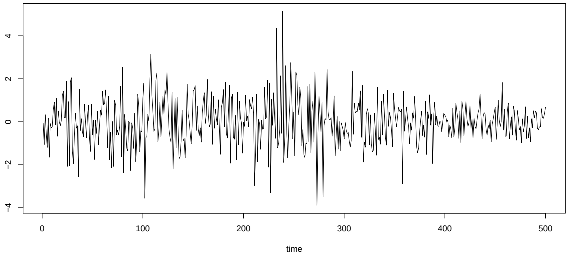
4. New state:

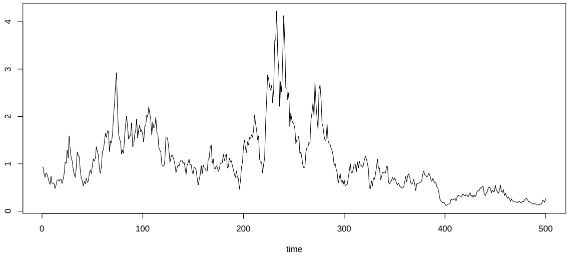
$$h_t^{(j+1)} = \begin{cases} h_t^* & \text{w. p. } \alpha \\ h_t^{(j)} & \text{w. p. } 1-\alpha \end{cases}$$

Example i. Simulated data

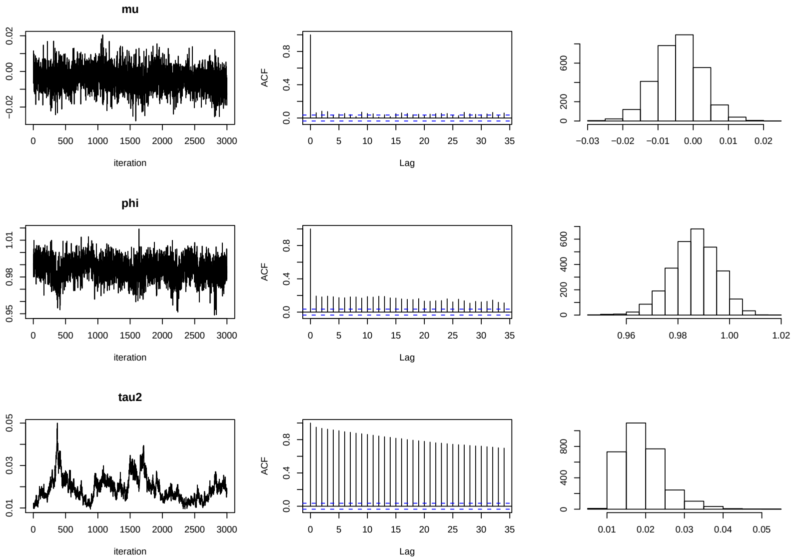
- Simulation setup
  - $n = 500$
  - $h_0 = 0.0$
  - $\mu = -0.00645$
  - $\phi = 0.99$
  - $\tau^2 = 0.15^2$
- Prior distribution
  - $\mu \sim N(0, 100)$
  - $\phi \sim N(0, 100)$
  - $\tau^2 \sim IG(10/2, 0.28125/2)$
  - $h_0 \sim N(0, 100)$
- MCMC setup
  - $M_0 = 1,000$
  - $M = 1,000$

Time series of y_t and $\exp\{h_t\}$

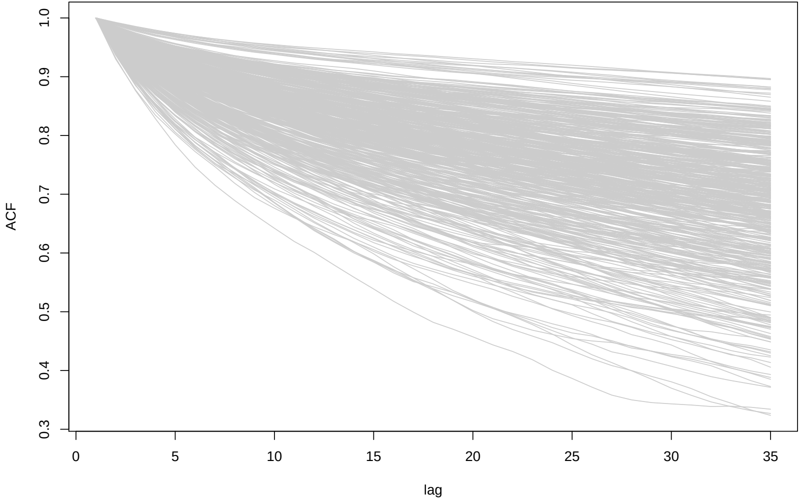




Parameters

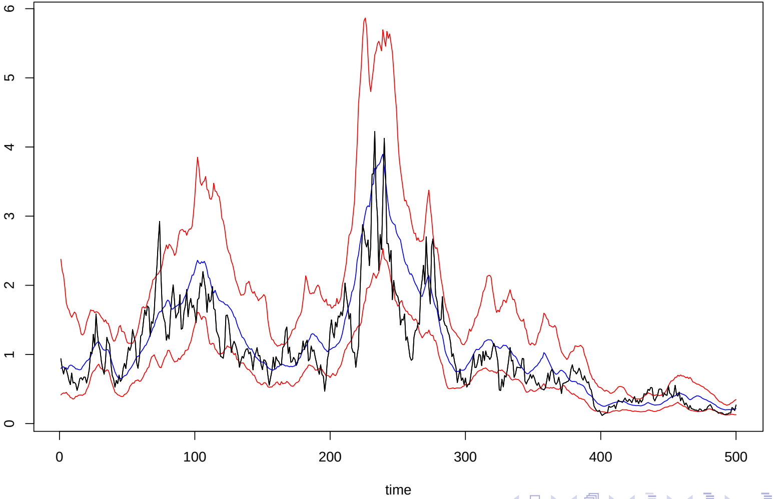

mu
iteration
ACF
Lag
phi
iteration
ACF
Lag
tau2
iteration
ACF
Lag

Autocorrelation of h_t



Volatilities

Tuning parameter: $v_h^2 = 0.01$



Sampling h_t via independent Metropolis-Hastings

The full conditional distribution of h_t is given by

$$
\begin{aligned}
p(h_t|h_{-t}, y^n, \theta, \tau^2) &= p(h_t|h_{t-1}, h_{t+1}, \theta, \tau^2)p(y_t|h_t) \\
&= f_N(h_t; \mu_t, \nu^2) f_N(y_t; 0, e^{h_t}).
\end{aligned}
$$

Kim, Shephard and Chib (1998) explored the fact that

$$
\log p(y_t|h_t) = \text{const} - \frac{1}{2}h_t - \frac{y_t^2}{2}\exp(-h_t)
$$

and that a Taylor expansion of $\exp(-h_t)$ around μ_t leads to

$$
\begin{aligned}
\log p(y_t|h_t) &\approx \text{const} - \frac{1}{2}h_t - \frac{y_t^2}{2}\left(e^{-\mu_t} - (h_t - \mu_t)e^{-\mu_t}\right) \\
g(h_t) &= \exp\left\{-\frac{1}{2}h_t(1 - y_t^2 e^{-\mu_t})\right\}
\end{aligned}
$$

Proposal distribution

Let $\nu_t^2 = \nu^2$ for $t = 1, \dots, n-1$ and $\nu_n^2 = \tau^2$.

Then, by combining $f_N(h_t; \mu_t, \nu_t^2)$ and $g(h_t)$, for $t = 1, \dots, n$, leads to the following proposal distribution:

$$q(h_t | h_{-t}, y^n, \theta, \tau^2) \equiv N\left(h_t; \tilde{\mu}_t, \nu_t^2\right)$$

where $\tilde{\mu}_t = \mu_t + 0.5\nu_t^2(y_t^2 e^{-\mu_t} - 1)$.

Metropolis-Hastings algorithm

For $t = 1, \ldots, n$

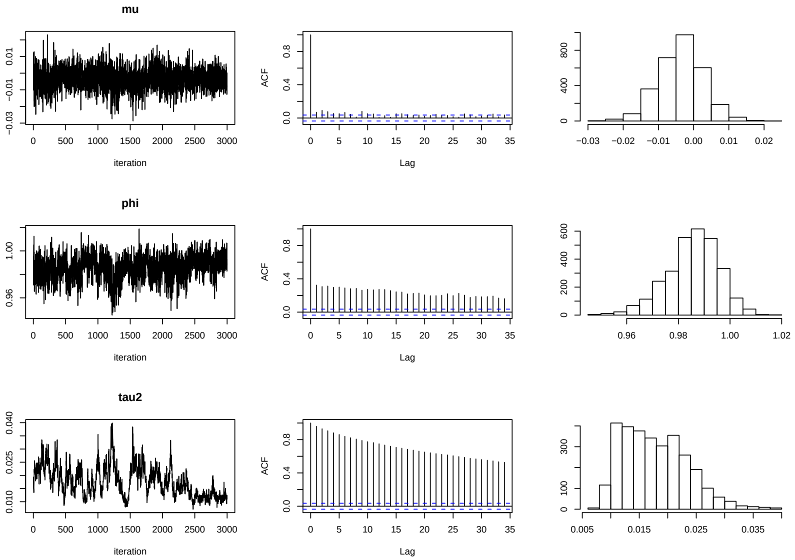
1. Current state: $h_t^{(j)}$
2. Sample h_t^* from $N(\tilde{\mu}_t, \nu_t^2)$
3. Compute the acceptance probability

$$\alpha = \min\left\{1, \frac{f_N(h_t^*; \mu_t, \nu_t^2) f_N(y_t; 0, e^{h_t^*})}{f_N(h_t^{(j)}; \mu_t, \nu_t^2) f_N(y_t; 0, e^{h_t^{(j)}})} \times \frac{f_N(h_t^{(j)}; \tilde{\mu}_t, \nu_t^2)}{f_N(h_t^*; \tilde{\mu}_t, \nu_t^2)}\right\}$$

4. New state:

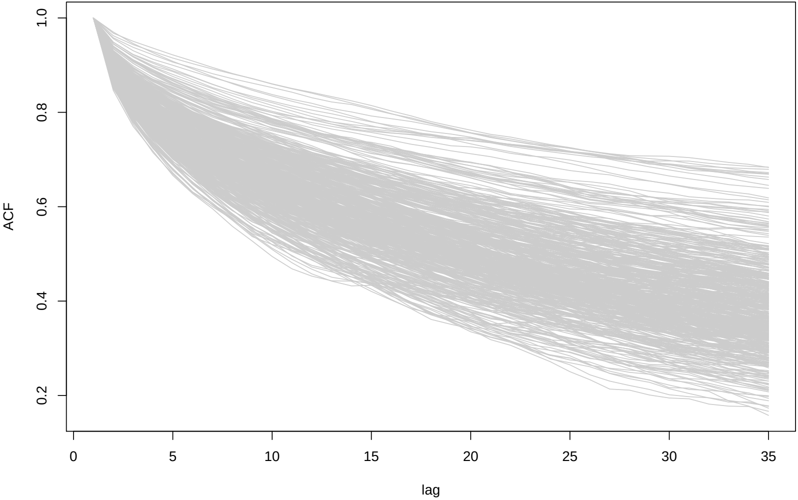
$$h_t^{(j+1)} = \begin{cases} h_t^* & \text{w. p. } \alpha \\ h_t^{(j)} & \text{w. p. } 1 - \alpha \end{cases}$$

Parameters

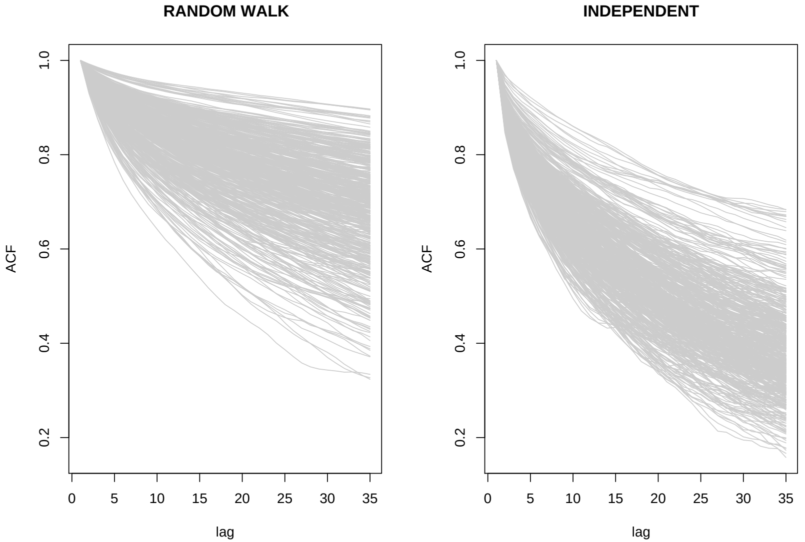


mu
phi
tau2
iteration
Lag
ACF
0 500 1000 1500 2000 2500 3000
0 5 10 15 20 25 30 35
−0.03 −0.02 −0.01 0.00 0.01 0.02
0.96 0.98 1.00 1.02
0.005 0.015 0.025 0.035

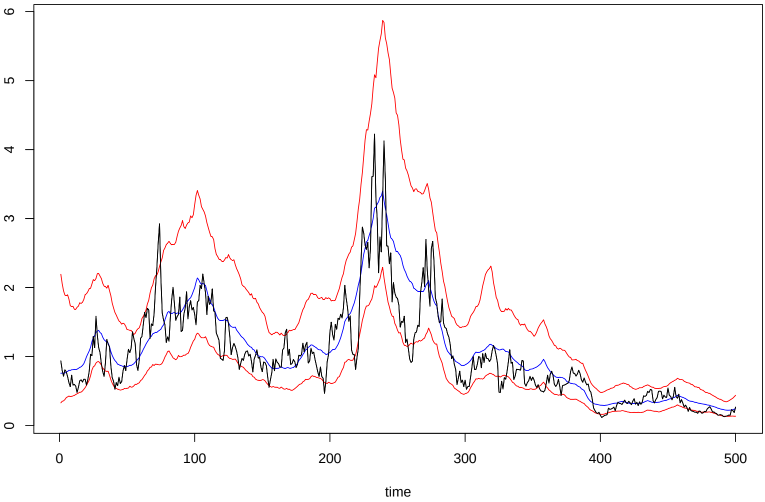
Autocorrelation of h_t



Autocorrelations of h_t for both schemes



Volatilities



Sampling h^n - normal approximation and FFBS

Let $y_t^* = \log y_t^2$ and $\epsilon_t = \log \varepsilon_t^2$.

The SV-AR(1) is a DLM with nonnormal observational errors, i.e.

$$\begin{aligned} y_t^* &= h_t + \epsilon_t \\ h_t &= \mu + \phi h_{t-1} + \tau \eta_t \end{aligned}$$

where $\eta_t \sim N(0,1)$.

The distribution of ϵ_t is $\log \chi_1^2$, where

$$\begin{aligned} E(\epsilon_t) &= -1.27 \\ V(\epsilon_t) &= \frac{\pi^2}{2} = 4.935 \end{aligned}$$

Normal approximation

Let ϵ_t be approximated by $N(\alpha, \sigma^2)$, $z_t = y_t^* - \alpha$, $\alpha = -1.27$ and $\sigma^2 = \pi^2/2$.

Then

$$\begin{aligned} z_t &= h_t + \sigma v_t \\ h_t &= \mu + \phi h_{t-1} + \tau \eta_t \end{aligned}$$

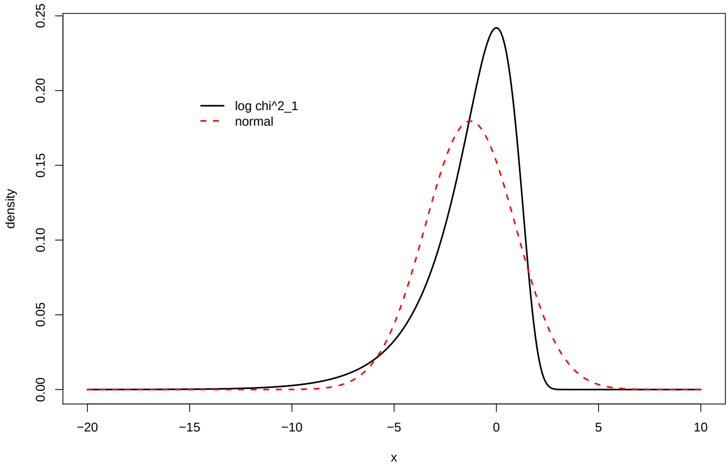
is a simple DLM where v_t and η_t are $N(0, 1)$.

Sampling from

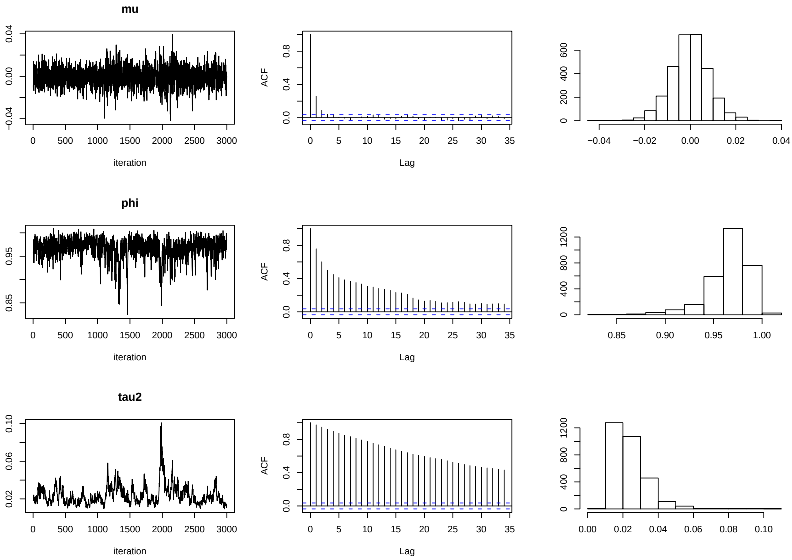
$$p(h^n | \theta, \tau^2, \sigma^2, z^n)$$

can be performed by the FFBS algorithm.

$\log \chi_1^2$ and $N(-1.27, \pi^2/2)$

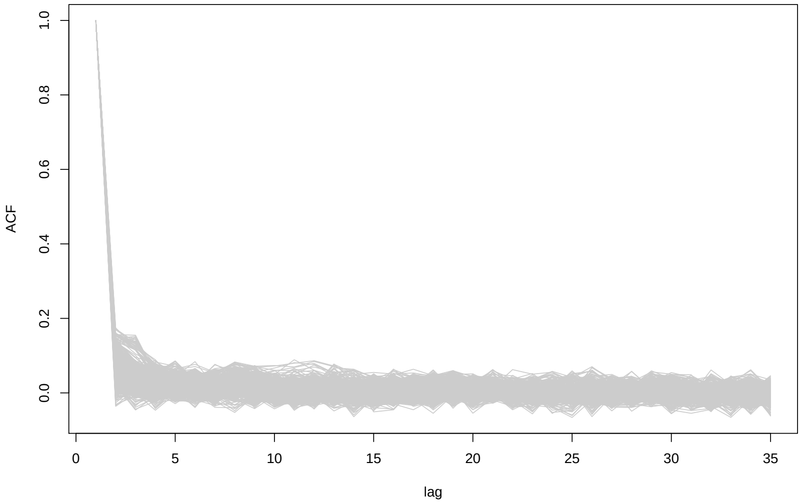


Parameters

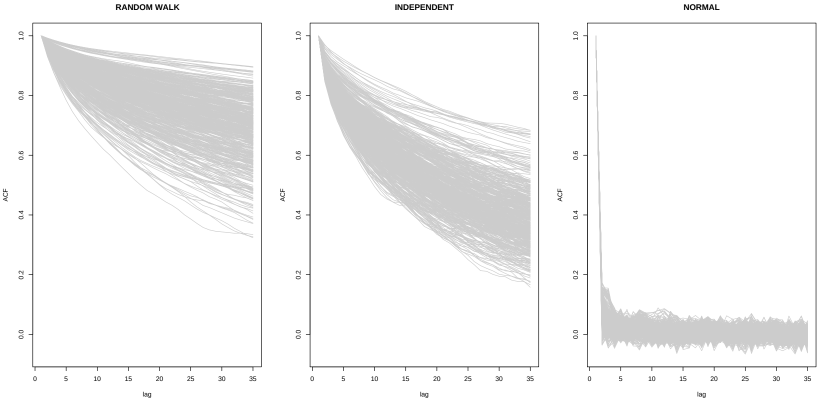


mu
0.04
0.00
-0.04
0 500 1000 1500 2000 2500 3000
iteration
ACF
0.0 0.4 0.8
0 5 10 15 20 25 30 35
Lag
0 200 400 600
−0.04 −0.02 0.00 0.02 0.04
phi
0.95
0.85
0 500 1000 1500 2000 2500 3000
iteration
ACF
0.0 0.4 0.8
0 5 10 15 20 25 30 35
Lag
0 400 800 1200
0.85 0.90 0.95 1.00
tau2
0.10
0.06
0.02
0 500 1000 1500 2000 2500 3000
iteration
ACF
0.0 0.4 0.8
0 5 10 15 20 25 30 35
Lag
0 400 800 1200
0.00 0.02 0.04 0.06 0.08 0.10

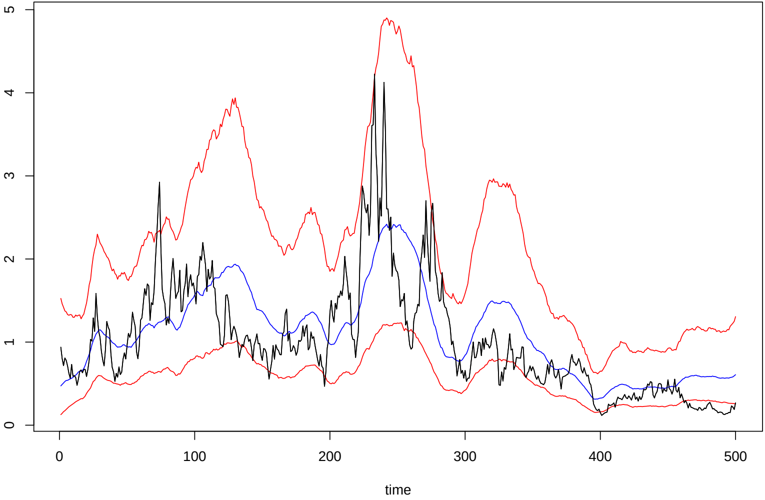
Autocorrelation of h_t



Autocorrelations of h_t for the three schemes



Volatilities



Sampling h^n - mixtures of normals and FFBS

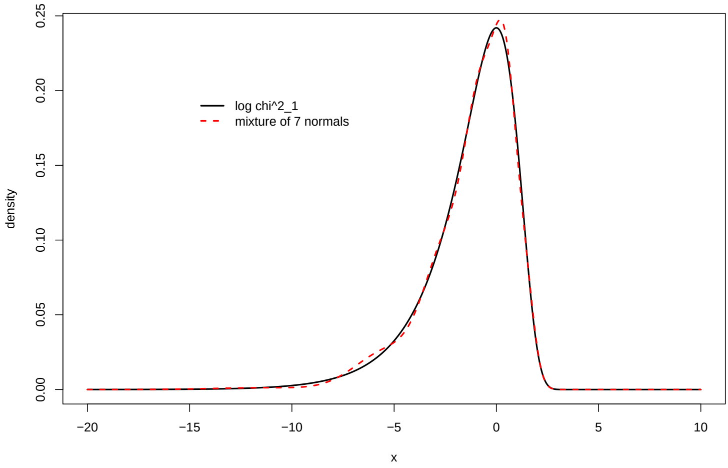
The log χ_1^2 distribution can be approximated by

$$\sum_{i=1}^{7} \pi_i N(\mu_i, \omega_i^2)$$

where

i	π_i	μ_i	ω_i^2
1	0.00730	-11.40039	5.79596
2	0.10556	-5.24321	2.61369
3	0.00002	-9.83726	5.17950
4	0.04395	1.50746	0.16735
5	0.34001	-0.65098	0.64009
6	0.24566	0.52478	0.34023
7	0.25750	-2.35859	1.26261

$\log \chi_1^2$ and $\sum_{i=1}^{7} \pi_i N(\mu_i, \omega_i^2)$



Mixture of normals

Using an argument from the Bayesian analysis of mixture of normal, let $z_1, \ldots, z_n$ be unobservable (latent) indicator variables such that $z_t \in \{1, \ldots, 7\}$ and $Pr(z_t = i) = \pi_i$, for $i = 1, \ldots, 7$.

Therefore, conditional on the z's, y_t is transformed into $\log y_t^2$,

$$\begin{aligned} \log y_t^2 &= h_t + \log \varepsilon_t^2 \\ h_t &= \mu + \phi h_{t-1} + \tau_\eta \eta_t \end{aligned}$$

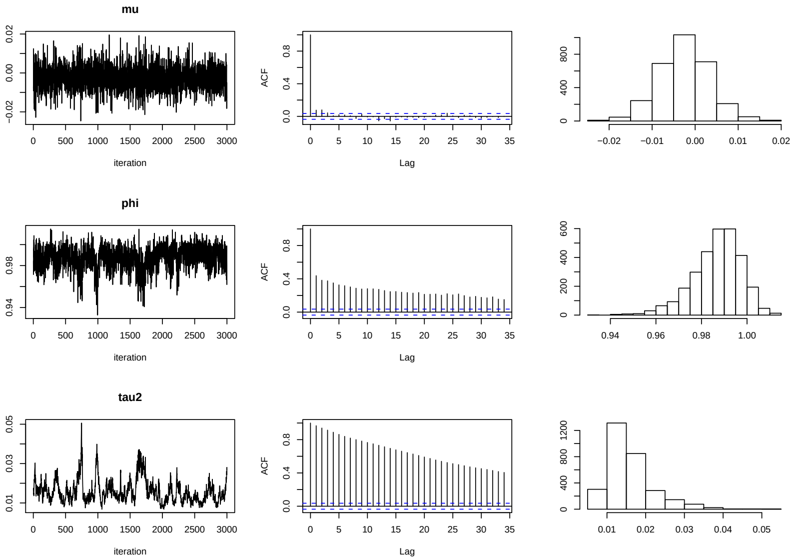
which can be rewritten as a normal DLM:

$$\begin{aligned} \log y_t^2 &= h_t + v_t & v_t \sim N(\mu_{z_t}, \omega_{z_t}^2) \\ h_t &= \mu + \phi h_{t-1} + w_t & w_t \sim N(0, \tau_\eta^2) \end{aligned}$$

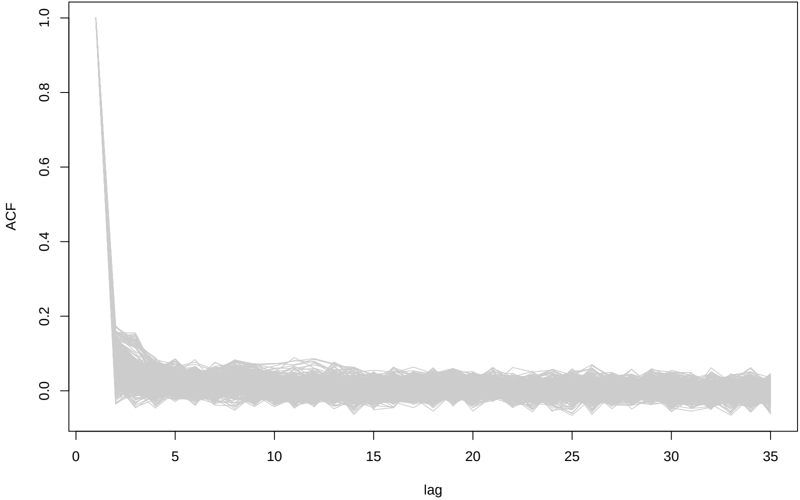
where μ_{z_t} and $\omega_{z_t}^2$ are provided in the previous table.

Then h^n is jointly sampled by using the the FFBS algorithm.

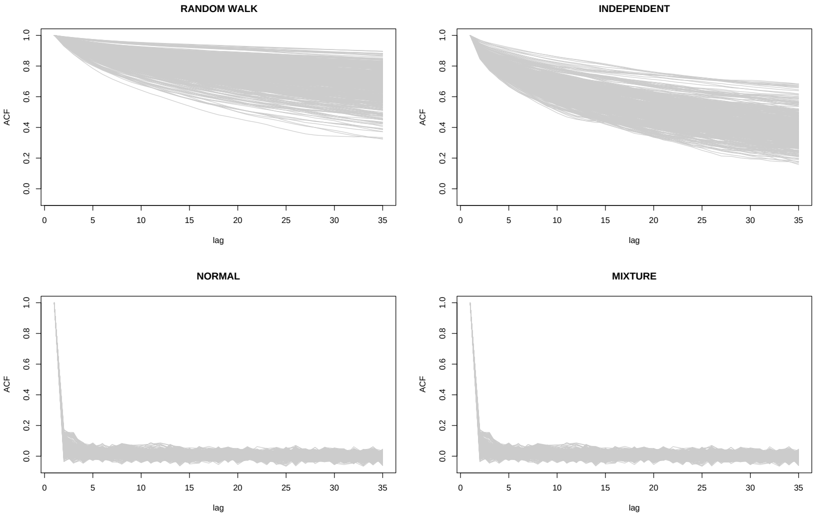
Parameters



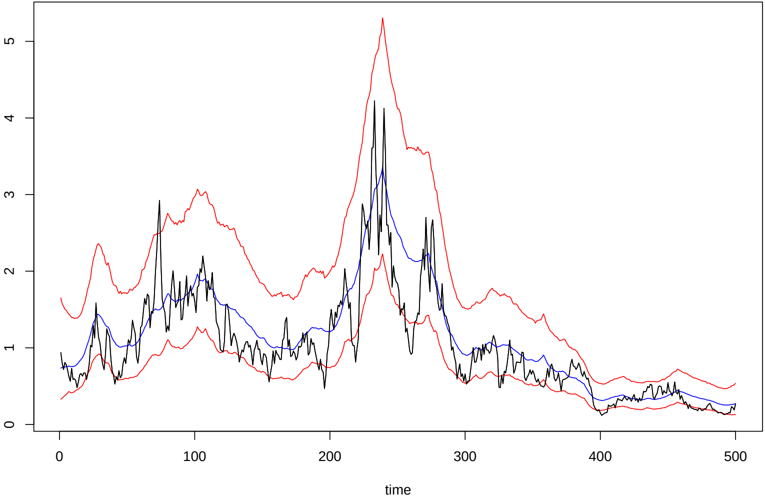
Autocorrelation of h_t



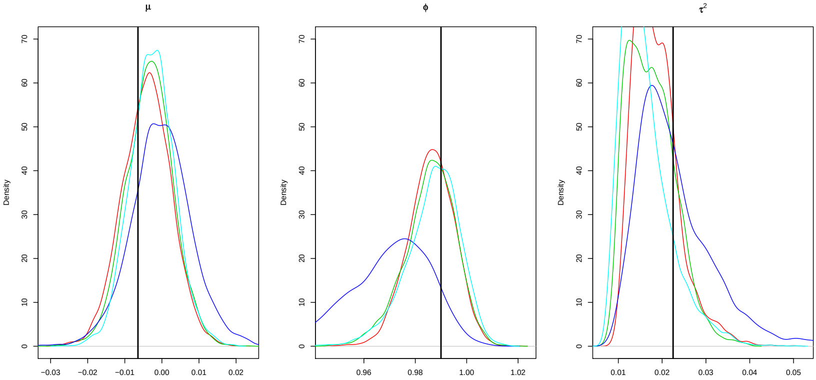
Autocorrelations of h_t for the four schemes



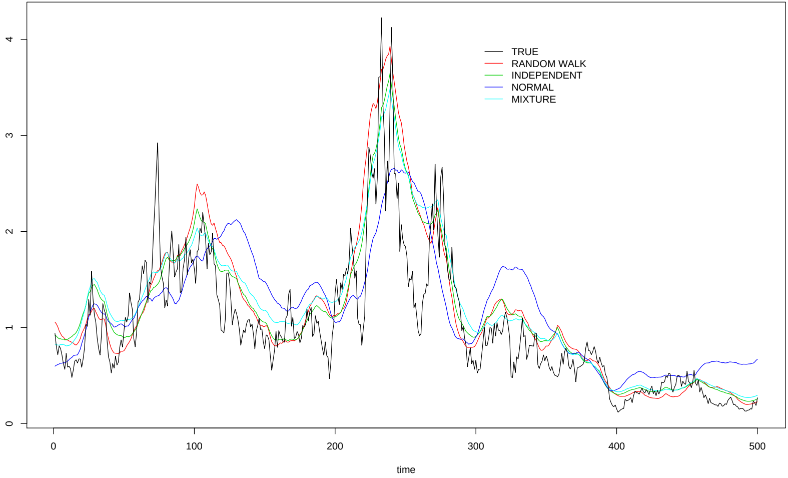
Volatilities



Comparing the four schemes: parameters



Comparing the four schemes: volatilities



Lopes and Salazar (2006)[1]

We extend the SV-AR(1) where

$$y_t \sim N(0, \exp\{h_t\})$$

to accommodate a smooth regime shift, i.e.

$$h_t \sim N(\alpha_{1t} + F(\gamma, \kappa, h_{t-d})\alpha_{2t}, \sigma^2)$$

where

$$\begin{aligned} \alpha_{it} &= \mu_i + \phi_i h_{t-1} + \delta_i h_{t-2} \qquad i = 1, 2 \\ F(\gamma, \kappa, h_{t-d}) &= \frac{1}{1 + exp\left(\gamma(\kappa - h_{t-d})\right)} \end{aligned}$$

such that $\gamma > 0$ drives smoothness and c is a threshold.

[1] Time series mean level and stochastic volatility modeling by smooth transition autoregressions: a Bayesian approach, In Fomby, T.B. (Ed.) *Advances in Econometrics: Econometric Analysis of Financial and Economic Time Series/Part B*, Volume 20, 229-242.

Modeling S&P500 returns

Data from Jan 7th, 1986 to Dec 31st, 1997 (3127 observations)

Models	AIC	BIC	DIC
AR(1)	12795	31697	7223.1
AR(2)	12624	31532	7149.2
LSTAR(1,d=1)	**12240**	**31165**	**7101.1**
LSTAR(1,d=2)	12244	31170	7150.3
LSTAR(2,d=1)	12569	31507	7102.4
LSTAR(2,d=2)	12732	31670	7159.4

	Models					
Parameter	$AR(1)$	$AR(2)$	LSTAR(1,1)	LSTAR(1,1)	LSTAR(2,1)	LSTAR(2,1)
	Posterior mean (standard deviation)					
μ_1	-0.060	-0.066	0.292	-0.354	-4.842	-6.081
	(0.184)	(0.241)	(0.579)	(0.126)	(0.802)	(1.282)
ϕ_1	0.904	0.184	0.306	0.572	-0.713	-0.940
	(0.185)	(0.242)	(0.263)	(0.135)	(0.306)	(0.699)
δ_1	-	0.715	-	-	-1.018	-1.099
		(0.248)			(0.118)	(0.336)
μ_2	-	-	-0.685	0.133	4.783	6.036
			(0.593)	(0.092)	(0.801)	(1.283)
ϕ_2	-	-	0.794	0.237	0.913	1.091
			(0.257)	(0.086)	(0.314)	(0.706)
δ_2	-	-	-	-	1.748	1.892
					(0.114)	(0.356)
γ	-	-	118.18	163.54	132.60	189.51
			(16.924)	(23.912)	(10.147)	(0.000)
κ	-	-	-1.589	0.022	-2.060	-2.125
			(0.022)	(0.280)	(0.046)	(0.000)
σ^2	0.135	0.234	0.316	0.552	0.214	0.166
	(0.020)	(0.044)	(0.066)	(0.218)	(0.035)	(0.026)

Carvalho and Lopes (2007)[3]

We extend the SV-AR(1) to accommodate a Markovian regime shift[2], i.e.

$$h_t \sim N(\mu_{s_t} + \phi h_{t-1}, \sigma^2)$$

and

$$Pr(s_t = j | s_{t-1} = i) = p_{ij} \qquad \text{for } i, j = 1, \ldots, k.x$$

for

$$\alpha_{s_t} = \gamma_1 + \sum_{j=1}^{k} \gamma_j I_{jt}$$

where $I_{jt} = 1$ if $s_t \geq j$ and zero otherwise, $\gamma_1 \in \Re$ and $\gamma_i > 0$ for $i > 1$.

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[2] So, Lam and Li (1998) A stochastic volatility model with Markov switching. *JBES*, 16, 244-253.

[3] Simulation-based sequential analysis of Markov switching stochastic volatility models, *Computational Statistics and Data Analysis*, 51, 4526-4542.

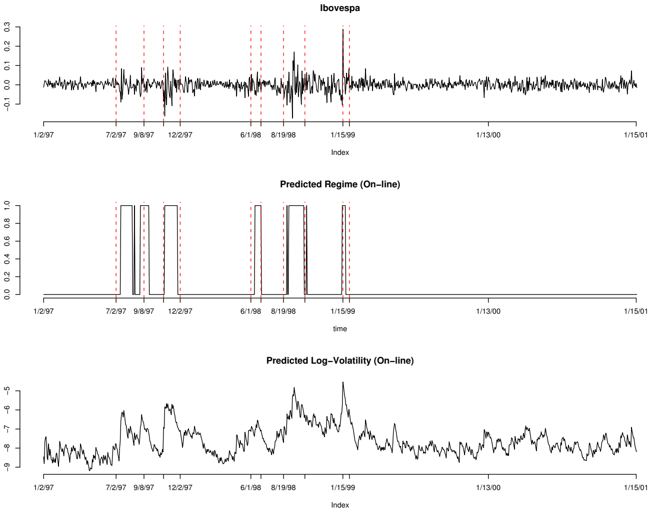
Modeling IBOVESPA returns

We analyzed IBOVESPA returns from 01/02/1997 to 01/16/2001 (1000 observations) based on a 2-regime model.

07/02/1997	Thailand devalues the baht by as much as 20%.
08/11/1997	IMF and Thailand set a rescue agreement.
10/23/1997	Hong Kong's stock index falls 10.4%. South Korea Won starts to weaken.
12/02/1997	IMF and South Korea set a bailout agreement.
06/01/1998	Russia's stock market crashes.
06/20/1998	IMF gives final approval to a loan package to Russia.
08/19/1998	Russia officially falls into default.
10/09/1998	IMF and World Bank joint meeting to discuss the global economic crisis. The Fed cuts interest rates.
01/15/1999	The Brazilian government allows its currency, the real, to float freely by lifting exchange controls.
02/02/1999	Arminio Fraga is named president of Brazil's Central Bank.

Model	95% credible interval	$E(\phi \mid D_T)$
SV	(0.9325;0.9873)	0.9525
MSSV	(0.8481;0.8903)	0.8707

Also, $E(p_{11} \mid D_T) = 0.993$ and $E(p_{11} \mid D_T) = 0.964$.


Ibovespa
-0.1 0.0 0.1 0.2 0.3
1/2/97 7/2/97 9/8/97 12/2/97 6/1/98 8/19/98 1/15/99 1/13/00 1/15/01
Index
Predicted Regime (On-line)
0.0 0.2 0.4 0.6 0.8 1.0
1/2/97 7/2/97 9/8/97 12/2/97 6/1/98 8/19/98 1/15/99 1/13/00 1/15/01
time
Predicted Log-Volatility (On-line)
-9 -8 -7 -6 -5
1/2/97 7/2/97 9/8/97 12/2/97 6/1/98 8/19/98 1/15/99 1/13/00 1/15/01
Index

Abanto, Migon and Lopes (2009)[4]

We use a modied mixture model with Markov switching volatility specfication to analyze the relationship between stock return volatility and trading volume, i.e.

$$
\begin{aligned}
y_t|h_t &\sim t_\nu(0, \exp\{h_t\}) \\
v_t|h_t &\sim Poisson(m_0 + m_1 \exp\{h_t\}) \\
h_t &\sim N(\mu + \gamma s_t + \phi h_{t-1}, \tau^2)
\end{aligned}
$$

with $s_t = 0$ or $s_t = 1$, $\mu \in R$ and $\gamma < 0$.

[4] Bayesian modeling of financial returns: a relationship between volatility and trading volume. *Applied Stochastic Models in Business and Industry*. Available online since June 8th 2009.

Lopes and Polson (2010)[5]

The *stochastic volatility with correlated jumps* (SVCJ) model of Eraker, Johannes and Polson (2003) can be written as

$$\begin{aligned} y_{t+1} &= y_t + \mu\Delta + \sqrt{v_t\Delta}\epsilon^y_{t+1} + J^y_{t+1} \\ v_{t+1} &= v_t + \kappa(\theta - v_t) + \sigma_v\sqrt{v_t\Delta}\epsilon^v_{t+1} + J^v_{t+1} \end{aligned}$$

where both ϵ^y_{t+1} and ϵ^v_{t+1} follow $N(0,1)$ with $\text{corr}(\epsilon^y_{t+1}, \epsilon^v_{t+1}) = \rho$; and jump components

$$\begin{aligned} J^y_{t+1} &= \xi^y_{t+1} N_{t+1} \qquad J^v_{t+1} = \xi^v_{t+1} N_{t+1} \\ \xi^v_{t+1} &\sim Exp(\mu_v) \\ \xi^y_{t+1} | \xi^v_{t+1} &\sim N(\mu_y + \rho_J \xi^v_{t+1}, \sigma^2_y) \\ Pr(N_{t+1} = 1) &= \lambda\Delta \end{aligned}$$

Usually, $\Delta = 1$.

[5] Extracting SP500 and NASDAQ volatility: The credit crisis of 2007-2008. *Handbook of Applied Bayesian Analysis*, (to appear).

Credit crisis of 2007

SV model: $\mu = J^y_{t+1} = J^v_{t+1} = 0$ and $\sqrt{v_t \Delta} = 1$ in the evolution equation.

SP500	Mean	StDev	2.5%	97.5%
$\kappa\theta$	-0.0031	0.0029	-0.0092	0.0022
$1-\kappa$	0.9949	0.0036	0.9868	1.0011
σ^2_v	0.0076	0.0026	0.0041	0.0144

SVJ model: $\mu = J^v_{t+1} = \xi^v_{t+1} = 0$ and $\sqrt{v_t \Delta} = 1$ in the evolution equation.

SP500	Mean	StDev	2.5%	97.5%
$\kappa\theta$	-0.0117	0.0070	-0.0262	0.0014
$1-\kappa$	0.9730	0.0084	0.9551	0.9886
σ^2_v	0.0432	0.0082	0.0302	0.0613
λ	0.0025	0.0017	0.0003	0.0066
μ_y	-2.7254	0.1025	-2.9273	-2.5230
σ^2_y	0.3809	0.2211	0.1445	0.9381

Lopes and Migon (2002) and Lopes and Carvalho (2007)[7]

The FSV model of Pitt and Shephard (1999) and Aguilar and West (2000)[6] can be written as

$$
\begin{aligned}
y_t|f_t &\sim N(\beta f_t, \Sigma_t) \qquad \Sigma_t = \text{diag}(\sigma_{1t}^2, \ldots, \sigma_{pt}^2) \\
f_t &\sim N(0, H_t) \qquad H_t = \text{diag}(\sigma_{p+1,t}^2, \ldots, \sigma_{p+k,t}^2) \\
\log(\sigma_{it}^2) = \eta_{it} &\sim N(\alpha_i + \gamma_i \eta_{i,t-1}, \xi_i^2) \qquad i = 1, \ldots, p \\
\log(\sigma_{jt}^2) = \lambda_{jt} &\sim N(\mu_j + \phi_j \lambda_{j,t-1}, \tau_j^2) \qquad j = 1, \ldots, k \\
\beta_{ijt} &\sim N(\zeta_{ij} + \Theta_{ij} \beta_{ij,t-1}, \omega_{ij}^2)
\end{aligned}
$$

for $i = 2, \ldots, p$ and $j = 1, \ldots, \min(i-1, k)$.

[6] AW: Bayesian dynamic factor models and variance matrix discounting for portfolio allocation. *JBES*, 18, 338357. PS: Time varying covariances: a factor stochastic volatility approach (with discussion). *Bayesian Statistics, Volume 6*, 547-570.

[7] LM:Comovements and contagion in emergent markets: stock indexes volatilities. *Case Studies in Bayesian Statistics*, Volume VI, 285-300. LC: Factor stochastic volatility with time varying loadings and Markov switching regimes. *Journal of Statistical Planning and Inference*, 137, 3082-3091.

Daily exchange rate

Returns on weekday closing spot prices for six currencies relative to the US dollar.

The data span the period from 1/1/1992 to 10/31/1995 inclusive.

- German Mark(DEM)
- British Pound(GBP)
- Japanese Yen(JPY)
- French Franc(FRF)
- Canadian Dollar(CAD)
- Spanish Peseta(ESP)

A $k = 3$ factor stochastic volatility model with time-varying loadings was implemented with relatively vague priors for all model parameters.

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