Homework 1

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Course: STP 598 - Time Series (Class # 23811)
Semester: Spring 2023
Due date: 10:30am January 31st, 2023 (via a single PDF file to hedibert@gmail.com)

1. **Moving average filter**

   (a) Generate $n = 100$ observations from the autoregression

   $$x_t = -0.9x_{t-1} + \omega_t$$

   with ω_t following a Gaussian white noise with standard deviation $\sigma_\omega = 1$ and $x_0 = 0$. Next, apply the moving average filter

   $$v_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$$

   to x_t, the data you generated. Now plot x_t as a line and superimpose v_t as a dashed line. Comment on the behavior of x_t and how applying the moving average filter changes that behavior.

   (b) Repeat (a) but with $x_t = cos(2\pi t)$.

   (c) Repeat (b) but with added $N(0, 1)$ noise, $x_t = cos(2\pi t) + \omega_t$.

   (d) Compare and contrast (a)-(c).

2. **Trend-stationarity**
   Consider the time series

   $$x_t = \beta_1 + \beta_2 t + \omega_t,$$

   where β_1 and β_2 are known constants and ω_t is a white noise process with variance σ_ω^2.

   (a) Determine whether x_t is stationary.

   (b) Show that the process $y_t = x_t - x_{t-1}$ is stationary.

   (c) Show that the mean of the moving average

   $$v_t = \frac{1}{2q+1} \sum_{j=-q}^{q} x_{t-j}$$

   is $\beta_1 + \beta_2 t$, and give a simplified expression for the autocovariance function.

3. **Random walk with drift model**
   Consider the random walk with drift model

   $$x_t = \delta + x_{t-1} + \omega_t,$$

   for $t = 1, 2, \ldots$, with $x_0 = 0$, where ω_t is white noise with variance σ_ω^2.

   (a) Show that the model can be written as $x_t = \delta t + \sum_{k=1}^{t} \omega_k$.

   (b) Find the autocovariance function of x_t.

   (c) Argue that x_t is not stationary.

   (d) Suggest a transformation to make the series stationary, and prove that the transformed series is stationary. (Hint: See Problem 2(b))

4. **Real data exercise**
In this exercise you will fit AR models to two well known time series from the US economy: i) Quarterly real GDP growth rates and ii) Quarterly real nondurables consumption growth rates. The following R script will help you start your analysis. However, you need to download the data from the zip file available in the URLs provided within the R script

```
# https://www.ssc.wisc.edu/~bhansen/econometrics
# https://www.ssc.wisc.edu/~bhansen/econometrics/Econometrics%20Data.zip
# Book source: Econometrics, by Bruce E. Hansen, 2022

library(readxl)
FRED_QD = read_excel("FRED-QD.xlsx")
attach(FRED_QD)
n = nrow(FRED_QD)

# y1: Quarterly real GDP growth rates
# y2: Quarterly real nondurables consumption growth rates

# Computing growth rates
y1 = 100*(gdpc1[2:n]/gdpc1[1:(n-1)]-1)
y2 = 100*(pcndx[2:n]/pcndx[1:(n-1)]-1)

# this is the new sample size, since one observation
# is lost when computing growth rates.
n = n-1

par(mfrow=c(2,2))
plot.ts(y1,ylab="",main="Real GDP growth rates")
acf(y1,main="")
plot.ts(y2,ylab="",main="Real nondurables consumption\ngrowth rates")
acf(y2,main="")
```

(a) Plot the time series and their empirical ACF.

(b) Fit $AR(1), \ldots, AR(5)$ to both time series y_1 and y_2.

(c) Compare the models and chose the lag that minimizes AIC within the fitted data.

(d) *Out-of-sample comparison:* Fit the AR models for both time series using the data up to time $t_0 = 200$, then compute h-step-ahead forecasts, for $h = 1, 4, 8, 12$. Then, repeat this for $t_0 = 201, 202, \ldots, 223$. We can now compute an average h-step-ahead MSE , i.e. MSE(1),MSE(4),MSE(8) and MSE(12), based on 24 observations (quarters). Comment in details your findings.