Model:

$$y_t = \begin{cases} \phi_1 y_{t-1} + \epsilon_t & t \leq \tau \\ \phi_2 y_{t-1} + \epsilon_t & t > \tau \end{cases} \quad \epsilon_t \overset{iid}{\sim} N(0, \sigma^2) \quad t = 2, \dots, n$$

Prior:

$$p(\phi_1, \phi_2, \sigma^2, \tau) = p(\phi_1) p(\phi_2) p(\sigma^2) p(\tau)$$

$$\phi_1, \phi_2 \overset{iid}{\sim} N(m_0, C_0)$$

$$\sigma^2 \sim IG(a_0, b_0)$$

$$\tau \sim \mathcal{U}\{t_0, \dots, t_1)$$

Full conditionals:

$$p(\phi_1 | y_{1:n}, \phi_2, \sigma^2, \tau) \propto p(\phi_1) \prod_{t=2}^{\tau} p_N(y_t | \phi_1 y_{t-1}, \sigma^2)$$

$$\propto \exp\left\{-\frac{1}{2C_0}(\phi_1^2 - 2\phi_1 m_0)\right\} \prod_{t=2}^{\tau} \exp\left\{-\frac{1}{2\sigma^2}(\phi_1^2 y_{t-1}^2 - 2\phi_1 y_{t-1} y_t)\right\}$$

$$\propto \exp\left\{-\frac{1}{2}\left[\phi_1^2\left(\frac{1}{C_0} + \frac{\sum_{t=2}^{\tau} y_{t-1}^2}{\sigma^2}\right) - 2\phi_1\left(\frac{m_0}{C_0} + \frac{\sum_{t=2}^{\tau} y_t y_{t-1}}{\sigma^2}\right)\right]\right\}$$

$$\therefore (\phi_1 | y_{1:n}, \sigma^2, \tau) \sim N\left[\left(\frac{1}{C_0} + \frac{\sum_{t=2}^{\tau} y_{t-1}^2}{\sigma^2}\right)^{-1}\left(\frac{m_0}{C_0} + \frac{\sum_{t=2}^{\tau} y_t y_{t-1}}{\sigma^2}\right); \left(\frac{1}{C_0} + \frac{\sum_{t=2}^{\tau} y_{t-1}^2}{\sigma^2}\right)\right]$$

Similarly,

$$(\phi_2 | y_{1:n}, \sigma^2, \tau) \sim N\left[\left(\frac{1}{C_0} + \frac{\sum_{t=\tau+1}^{n} y_{t-1}^2}{\sigma^2}\right)^{-1}\left(\frac{m_0}{C_0} + \frac{\sum_{t=\tau+1}^{n} y_t y_{t-1}}{\sigma^2}\right); \left(\frac{1}{C_0} + \frac{\sum_{t=\tau+1}^{n} y_{t-1}^2}{\sigma^2}\right)\right]$$

$$p(\sigma^2 | y_{1:n}, \phi_1, \phi_2, \tau) \propto (\sigma^2)^{-(a_0+1)} e^{-b_0/\sigma^2}$$

$$\times (\sigma^2)^{-\frac{n-1}{2}} e^{-\frac{\sum_{t=2}^{\tau}(y_t - \phi_1 y_{t-1})^2 + \sum_{t=\tau+1}^{n}(y_t - \phi_2 y_{t-1})^2}{2\sigma^2}}$$

$$\therefore (\sigma^2 | y_{1:n}, \phi_1, \phi, \tau) \sim IG\left(a_0 + \frac{n-1}{2} ; b_0 + \frac{\sum_{t=2}^{\tau}(y_t - \phi_1 y_{t-1})^2 + \sum_{t=\tau+1}^{n}(y_t - \phi_2 y_{t-1})^2}{2}\right)$$

$$p(\tau | y_{1:n}, \phi_1, \phi_2, \sigma^2) \propto \prod_{t=2}^{\tau} p_N(y_t | \phi_1 y_{t-1}, \sigma^2) \prod_{t=\tau+1}^{n} p_N(y_t | \phi_2 y_{t-1}, \sigma^2)$$

for $\tau \in \{t_0, \ldots, t_1\}$.