Homework 1

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Course: STP 598 - Time Series (Class # 12767)
Semester: Spring 2022
Due date: 12pm, February 14th, 2022.

1. **Moving average filter**

   (a) Generate $n = 100$ observations from the autoregression

   $$x_t = -0.9x_{t-1} + \omega_t$$

   with ω_t following a Gaussian white noise with standard deviation $\sigma_\omega = 1$ and $x_0 = 0$. Next, apply the moving average filter

   $$v_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$$

   to x_t, the data you generated. Now plot x_t as a line and superimpose v_t as a dashed line. Comment on the behavior of x_t and how applying the moving average filter changes that behavior.

   (b) Repeat (a) but with $x_t = cos(2\pi t)$.

   (c) Repeat (b) but with added $N(0,1)$ noise, $x_t = cos(2\pi t) + \omega_t$.

   (d) Compare and contrast (a)-(c).

2. **Trend-stationarity**
   Consider the time series

   $$x_t = \beta_1 + \beta_2 t + \omega_t,$$

   where β_1 and β_2 are known constants and ω_t is a white noise process with variance σ^2_ω.

   (a) Determine whether x_t is stationary.

   (b) Show that the process $y_t = x_t - x_{t-1}$ is stationary.

   (c) Show that the mean of the moving average

   $$v_t = \frac{1}{2q+1} \sum_{j=-q}^{q} x_{t-j}$$

   is $\beta_1 + \beta_2 t$, and give a simplified expression for the autocovariance function.

3. **Random walk with drift model**
Consider the random walk with drift model

$$x_t = \delta + x_{t-1} + \omega_t,$$

for $t = 1, 2, \ldots$, with $x_0 = 0$, where ω_t is white noise with variance σ_ω^2.

   (a) Show that the model can be written as $x_t = \delta t + \sum_{k=1}^{t} \omega_k$.
   (b) Find the autocovariance function of x_t.
   (c) Argue that x_t is not stationary.
   (d) Suggest a transformation to make the series stationary, and prove that the transformed series is stationary. (Hint: See Problem 2(b))

4. **Moving-average process**
Suppose that the simple return of a monthly bond index follows the $MA(1)$ model

$$x_t = \omega_t + 0.2\omega_{t-1},$$

where ω_t is a white noise process with standard deviation $\sigma_\omega = 0.025$. Assume that $\omega_{100} = 0.01$. Compute the 1-step- and 2-step-ahead forecasts of the return at the forecast origin $t = 100$. What are the standard deviations of the associated forecast errors? Also compute the lag-1 and lag-2 autocorrelations of the return series.

5. **Autoregressive process**
Suppose that the daily log return of a security follows the model

$$x_t = 0.01 + 0.2x_{t-2} + \omega_t$$

where ω_t is a Gaussian white noise series with mean zero and variance $\sigma_\omega^2 = 0.02$. What are the mean and variance of the return series x_t? Compute the lag-1 and lag-2 autocorrelations of x_t. Assume that $x_{100} = -0.01$, and $x_{99} = 0.02$. Compute the 1- and 2-step-ahead forecasts of the return series at the forecast origin $t = 100$. What are the associated standard deviations of the forecast errors?