Dynamic Ordering Learning in Multivariate Forecasting

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Overview

Introduction

Model uncertainty is a well-known challenge in many areas such as engineering signal processing, neuroscience and financial econometrics.

Specially when the main goal is to produce sequential forecasts to decision-making problems.

For many applications, we don't know exactly the:

- main predictors to choose for each period t;
- time-variation in coefficients and volatilities;
- degree of time-variation of coefficients over time.

Introduction

Recent literature has used the notion of Bayesian model selection/average among different models.

Univariate Case: Raftery, Kárnỳ, and Ettler (2010), Koop and Korobilis (2012), Dangl and Halling (2012), Catania, Grassi, and Ravazzolo (2019) and Levy and Lopes (2020).

Multivariate Case: Koop and Korobilis (2013), Koop and Korobilis (2014) and Beckmann, Koop, Korobilis, and Schüssler (2020).

Advances in the literature

Both methods propose analytical solutions:

- Wishart DLM (W-DLM): Two restrictions: it forces all the time series in the system to share the same vector of predictors, and second, variances and covariances are modeled in the same structure and must time-varying jointly.
- Dynamic Dependency Network Models (DDNM): Each time series can feature its own set of predictor variables and allow for separate degrees of time-variation for variances and covariances.

Advances in the literature

Dynamic Dependency Network Models (DDNM): Zhao, Xie, and West (2016), West (2020), Fisher, Pettenuzzo, Carvalho, et al. (2020) and Lavine, Lindon, West, et al. (2020).

DDNM defines multivariate dynamic models via coupling of sets of customized univariate DLMs (Decouple/Recouple).

The structure is related to the popular Cholesky-style Multivariate SV (Primiceri, 2005; Lopes et al., 2018).

Cholesky-style models: depend on series orderings!

Econometric Framework

Consider $\mathbf{y_t}$ as a m-dimensional vector with time series $y_{j,t}$ and consider the following dynamic system:

$$(\mathbf{I}_m - \Gamma_t)\,\mathbf{y}_t = \begin{pmatrix} \mathbf{x}'_{1,t-1}\boldsymbol{\beta}_{1t} \\ \vdots \\ \mathbf{x}'_{m,t-1}\boldsymbol{\beta}_{mt} \end{pmatrix} + \boldsymbol{\nu}_\mathbf{t}, \qquad \boldsymbol{\nu}_\mathbf{t} \mid \Omega_t \sim \mathcal{N}\left(\mathbf{0}, \Omega_t\right), \qquad (1)$$

- $\mathbf{x}_{j,t-1}$ is a p-dimensional vector of predictors;
- $\boldsymbol{\beta}_{jt}$ are time-varying coefficients;
- $\Omega_t = \text{diag}\left(\sigma^2_{1t}, \ldots, \sigma^2_{mt}\right)$;
- All contemporaneous relations come from the $m \times m$ matrix Γ_t.

Cholesky decomposition

Following the DDNM of Zhao et al. (2016), we focus on the particular case where $\Gamma_{\mathbf{t}}$ is lower triangular with zeroes in and above the main diagonal:

$$\Gamma_t = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 \\ \gamma_{21,t} & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \gamma_{m1,t} & \gamma_{m2,t} & \cdots & \gamma_{m,m-1,t} & 0 \end{bmatrix} \tag{2}$$

This lower diagonal structure has already appeared in the econometric literature (Lopes et al., 2018, Shirota, Omori, Lopes, and Piao, 2017, Carvalho, Lopes and McCulloch 2018, Primiceri, 2005 and others) and started to become known as a *Cholesky-style* framework.

Econometric Framework

Equation (1) can be rewritten in the reduced form as

$$\mathbf{y}_t = \mathbf{A}_t \begin{pmatrix} \mathbf{x}'_{1,t-1}\boldsymbol{\beta}_{1t} \\ \vdots \\ \mathbf{x}'_{m,t-1}\boldsymbol{\beta}_{mt} \end{pmatrix} + \mathbf{u}_t \quad \mathbf{u}_t \mid \Sigma_t \sim \mathcal{N}(0, \Sigma_t) \tag{3}$$

where $\mathbf{A}_t = (\mathbf{I}_m - \Gamma_t)^{-1}$ and $\mathbf{u}_t = \mathbf{A}_t \boldsymbol{\nu}_\mathbf{t}$.

- The modified Cholesky decomposition clearly appears in $\Sigma_t = \mathbf{A}_t \Omega_t \mathbf{A}'_t$ which is now a full variance-covariance matrix.
- Given the parental triangular structure of $\Gamma_\mathbf{t}$ in (2), the equations will be conditionally independent.
- The model can be viewed as a set of m conditionally independent univariate DLMs.

m univariate DLMs

The set of m univariate models can be represented as m univariate recursive dynamic regressions, for $j = 1, \ldots, m$:

$$y_{jt} = \mathbf{x}'_{j,t-1}\boldsymbol{\beta}_{jt} + \mathbf{y}'_{<j,t}\boldsymbol{\gamma}_{<j,t} + \nu_{jt}, \qquad \nu_{jt} \sim \mathcal{N}\left(0, \sigma^2_{jt}\right), \tag{4}$$

and dynamic coefficients evolving according to random walks:

$$\begin{pmatrix} \boldsymbol{\beta}_{jt} \\ \boldsymbol{\gamma}_{<j,t} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\beta}_{j,t-1} \\ \boldsymbol{\gamma}_{<j,t-1} \end{pmatrix} + \boldsymbol{\omega}_{jt} \quad \boldsymbol{\omega}_{jt} \sim \mathcal{N}\left(\mathbf{0}, \mathbf{W}_{jt}\right). \tag{5}$$

m univariate DLMs

By defining the full dynamic state and regression vectors as

$$\boldsymbol{\theta}_{jt} = \begin{pmatrix} \boldsymbol{\beta}_{jt} \\ \boldsymbol{\gamma}_{<j,t} \end{pmatrix} \quad \text{and} \quad \mathbf{F}_{jt} = \begin{pmatrix} \mathbf{x}_{j,t-1} \\ \mathbf{y}_{<j,t} \end{pmatrix},$$

we recover the traditional univariate DLM formulation as in West and Harrison (1997), namely

$$\begin{aligned} y_{jt} &= \mathbf{F}'_{jt}\boldsymbol{\theta}_{jt} + \nu_{jt}, \qquad \nu_{jt} \sim N\left(0, \sigma^2_{jt}\right), \\ \boldsymbol{\theta}_{jt} &= \boldsymbol{\theta}_{j,t-1} + \boldsymbol{\omega}_{jt}, \qquad \boldsymbol{\omega}_{jt} \sim N\left(\mathbf{0}, \mathbf{W}_{jt}\right), \end{aligned}$$

for $j = 1, \ldots, m$, where again the evolution of β_{jt} and γ_{jt} evolve over time as a simple random-walk.

Conjugate Analysis

Posterior at $t-1$. Following West and Harrison (1997), Chapter 4, at time $t-1$ and for each time series j, the joint posterior distribution of $\boldsymbol{\theta}_{jt-1}$ and σ_{jt-1} at time $t-1$ is a multivariate Normal-Gamma:

$$\boldsymbol{\theta}_{j,t-1}, \sigma_{jt-1}^{-1} \mid \mathcal{D}_{t-1} \sim \mathcal{NG}\left(\mathbf{m}_{j,t-1}, \mathbf{C}_{j,t-1}, n_{j,t-1}, n_{j,t-1}s_{j,t-1}\right). \quad (6)$$

Through the random walk evolution and conjugacy, we can derive the joint prior distribution of $\boldsymbol{\theta}_{jt}$ and σ_{jt} for time t as:

$$\boldsymbol{\theta}_{jt}, \sigma_{jt}^{-1} \mid \mathcal{D}_{t-1} \sim \mathcal{NG}\left(\mathbf{a}_{jt}, \mathbf{R}_{jt}, r_{jt}, r_{jt}s_{j,t-1}\right) \quad (7)$$

where $r_{jt} = \kappa_j n_{j,t-1}$, $\mathbf{a}_{jt} = \mathbf{m}_{j,t-1}$ and $\mathbf{R}_{jt} = \mathbf{C}_{j,t-1}/\delta_j$. Here, we use $0 < \delta_j \leq 1$ and $0 < \kappa_j \leq 1$ as discount factors to induce time-variation in the evolution of parameters.

Conjugate Analysis

1-step ahead forecast at $t-1$. The (prior) predictive distribution of y_{jt} is a Student's t distribution with r_{jt} degrees of freedom:

$$y_{jt} \mid \mathbf{y}_{<j,t}, \mathcal{D}_{t-1} \sim \mathcal{T}_{r_{jt}}\left(f_{jt}, q_{jt}\right),$$

with $f_{jt} = \mathbf{F}'_{jt}\mathbf{a_{jt}}$ and $q_{jt} = s_{j,t-1} + \mathbf{F}'_{jt}\mathbf{R}_{jt}\mathbf{F}_{jt}$.

- Conjugate analysis for forward filters and one-step ahead forecasting.
- Closed-form solution for predictive densities for each equation j.

Conditional on *parents*, the joint predictive density for $\mathbf{y_t}$ is:

$$p\left(\mathbf{y}_t \mid \mathcal{D}_{t-1}\right) = \prod_{j=1}^{m} p\left(y_{jt} \mid \mathbf{y}_{<jt}, \mathcal{D}_{t-1}\right),. \tag{8}$$

which simply is the product of the m different univariate Student's t distributions.

Open Question and Contribution

DDNMs and the *Cholesky-style* framework require a specified order of the m series.

For some lower-dimensional series in macroeconomics, the ordering may reflect economic reasoning and theory.

However, in many cases, the dependency structure is uncertain.

Or the contemporaneous relations may evolve over time: the economic environment is always changing.

- We propose a highly flexible and fast method to deal with the problem of ordering uncertainty.
- NDLM closed-form sequential learning avoids the use of MCMC/SMC.

Contribution

We propose a dynamic method to deal with the uncertainty around series ordering and different contemporaneous dependencies across series in an online fashion.

Dynamic Ordering Selection/Average approach:
It can be applied in any field where the goal is to produce sequential forecasts for decision-making.

We show in two different applications how our econometric method outperforms some traditional benchmarks and the use of fixed order over time. The applications are:

- 1) Portfolio allocation;
- 2) Macroeconomic Forecasting.

Dynamic Ordering Learning

In Raftery et al. (2010) and Koop and Korobilis (2012), the model space is defined by different predictors and discount/forgetting factors.

Here, the models space will be also defined by different order structures.

Curse of dimensionality:
k time-series $\implies$ k! possible orders.

Dynamic Ordering Learning

We dynamically compute probabilities for each order.

For each period of time we can:

- select the best order (*dynamic order selection*, DOS), or
- average across all different orders (*dynamic order averaging*, DOA),

weighing by those order probabilities.

Similar to (8), we can compute the predictive density for each equation j at order i and then simply generate the joint predictive density for order i as:

$$p\left(\mathbf{y}_t \mid \mathcal{D}_{t-1}, \mathcal{O}_i\right) = \prod_{j=1}^{m} p\left(y_{jt} \mid \mathbf{y}_{<jt}, \mathcal{D}_{t-1}, \mathcal{O}_i\right). \tag{9}$$

Dynamic Ordering Learning

Each univariate model is conditionally independent given the parental set.

Given a specific order i:

- Known joint predictive distribution.
- The time series are decoupled for sequential analysis and then recoupled for forecasting into an optimal multivariate model.

Now, we are able to compute Dynamic Order Probabilities (DOP).

Dynamic Order Probabilibites

After computing the joint predictive density for all $k!$ orders, we just follow the laws of probability and compute the DOP. Let

$$\pi_{t-1|t-1,i} = p(\mathcal{O}_i \mid \mathcal{D}_{t-1}),$$

denote the posterior probability of order i at time $t-1$. The predicted probability of order i, given data until time $t-1$:

$$\pi_{t|t-1,i} = \frac{\pi^{\alpha}_{t-1|t-1,i}}{\sum_{l=1}^{K} \pi^{\alpha}_{t-1|t-1,l}}, \quad (10)$$

where $0 \leq \alpha \leq 1$ is a forgetting factor (Raftery et al., 2010). Then,

$$\pi_{t|t,i} = \frac{\pi_{t|t-1,i}\, p\left(\mathbf{y}_t \mid \mathcal{D}_{t-1}, \mathcal{O}_i\right)}{\sum_{l=1}^{K} \pi_{t|t-1,l}\, p\left(\mathbf{y}_t \mid \mathcal{D}_{t-1}, \mathcal{O}_l\right)}. \quad (11)$$

Predictors and discount factors learning

Similar to Raftery et al. (2010) and Koop and Korobilis (2012), we apply Dynamic Model Selection (DMS) for each equation in a given order structure.

We dynamically choose the univariate model with the best predictors and discount factors over time.

Since each equation is conditionally independent for a given order i:

$$P(\mathcal{M}^*_{1:m}|\mathcal{D}_{t-1}, \mathcal{O}_i) = \prod_{j=1}^{m} P(\mathcal{M}^*_j|\mathcal{D}_{t-1}, \mathcal{O}_i)$$

As soon as we select univariate models with the highest model probabilities in each order, we recover the best multivariate model for that specific ordering.

Our approach is also able to sequentially change predictors and discount factors for each equation.

Econometric Applications

We test how our approach performs in two different econometric contexts:

- 1) Portfolio Allocation
- 2) Macroeconomic Forecasting

We are interested in statistical (forecasting) and economic (portfolio) evaluation.

Statistical Evaluation

We make point (MSFE) and density (LPDR) forecast evaluations.

Mean Square Forecast Error (MSFE):

$$MSFE^{l} = \frac{\sum_{i=1}^{k} MSFE_i^{l}}{\sum_{i=1}^{k} MSFE_i^{Bmk}} \tag{12}$$

where l is the specific order to be evaluated and Bmk is the specific benchmark model.

Log-Predictive Density Ratio (LPDR):

$$\text{LPDR}_l = \sum_{t=1}^{T} \log\left\{ \frac{p_l\left(\mathbf{y_{t+1}} \middle| \mathbf{y_t}\right)}{p_{Bmk}\left(\mathbf{y_{t+1}} \middle| \mathbf{y_t}\right)} \right\}. \tag{13}$$

Economic Evaluation

For the portfolio allocation problem, we are not concerned about out-of-sample predictability, but how our approach improves final decisions and outcomes.

As in Fleming et al. (2001), Della Corte et al. (2009) and Beckmann et al. (2020), besides Sharp Ratios we also compute the performance fee (Φ) that an investor will be willing to pay to switch from the benchmark model to the DOL approach.

$$\sum_{t=0}^{T-1}\{(R_{p,t+1}^{DOL}-\Phi)-\gamma(R_{p,t+1}^{DOL}-\Phi)^2\}=\sum_{t=0}^{T-1}\{R_{p,t+1}^{Bmk}-\gamma(R_{p,t+1}^{Bmk})^2\}$$

where $\gamma = 0.5\theta/(1+\theta)$ and θ is the investor's degree of relative risk aversion and $R_{p,t}$ is the gross return of the portfolio at period t.

Portfolio Allocation Problem - Exchange Rates

Structural models have shown great difficulty to outperform a simple random walk model (Meese and Rogoff, 1983).

Growing literature on how to improve forecast performance:

Della Corte et al. (2009), Aastveit et al. (2018), Byrne et al. (2018) and Beckmann et al. (2020).

The environment of the economy is always changing: the relations between currencies may change and depend differently over time.

Portfolio Allocation Problem

Predictors: 12 measures of momentum. Look-Back period of 1 to 12 months.

Choose between constant or time-varying parameters: $\delta \in \{0.99, 1\}$ and $\kappa \in \{0.96, 1\}$.

The investor can adapt to a new forecasting environment each time period by switching to a new model, based on past forecast errors.

Monthly data from Beckmann et al. (2020) - from 1986:01 until 2016:12

Benchmark Model: Wishart Random-Walk (W-RW)

Dynamic Asset Allocation

We consider an US investor who builds a portfolio by allocating her wealth between 7 bonds: one domestic (US), and 6 foreign bonds.

In each period, the foreign bonds yield a riskless return in the local currency plus a risky return due to currency fluctuations.

Currencies: Australian dollar (AUD), the Canadian dollar (CAD), the Euro (EUR), the Japanese yen (JPY), the Swiss franc (SWF), the Great Britain pound (GBP) and the US dollar (USD).

Following Della Corte et. al (2009) and Byrne, Korobilis and Ribeiro (2018), the investor solves the following problem:

$$\begin{array}{c} \max_{w_t} \left\{ \mu_{p,t+1} = w_t' \mu_{t+1|t} + \left(1 - w_t' \iota\right) r_f \right\} \\ \text{s.t.} \quad \left(\sigma_p^*\right)^2 = w_t' \sum_{t+1|t} w_t \end{array} \tag{14}$$

Mean-Variance Investor

Solution:

$$w_t = \frac{\sigma_p^*}{\sqrt{C_t}} \sum_{t+1|t}^{-1} \left(\mu_{t+1|t} - \iota r_f\right)$$

with $C_t = \left(\mu_{t+1|t} - \iota r_f\right)' \sum_{t+1|t}^{-1} \left(\mu_{t+1|t} - \iota r_f\right)$.

For each period t,

- 1st step: Use the selected model to forecast one-period ahead returns and covariance matrix.
- 2nd step: Dynamically rebalance the portfolio by calculating the new optimal weights for each currency.

We consider a volatility target of $\sigma_p = 10\%$

Results

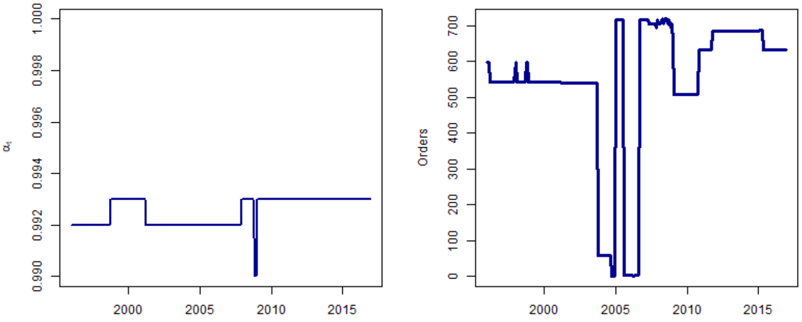

Figure: Time-Varying Forgetting Factor α_t (Left panel) and Order Selection (Right panel)

Statistical Evaluation

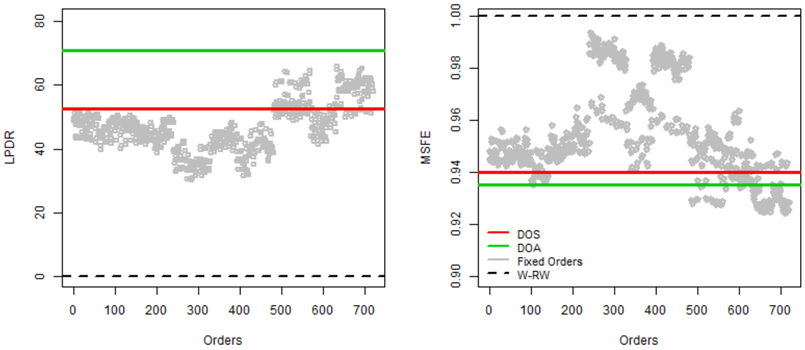

Figure: Statistical performance relative to the Wishart-Random-Walk model.
i) Left panel: Log Predictive Density Ratio (LPDR);
ii) Right panel: Mean Square Forecast Error (MSFE)

Statistical Evaluation

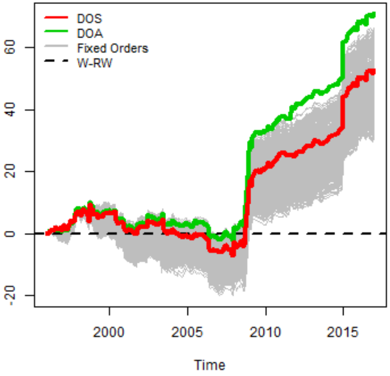

Figure: Accumulated Log Predictive Likelihood relative to the Wishart-Random Walk Model

Economic Evaluation

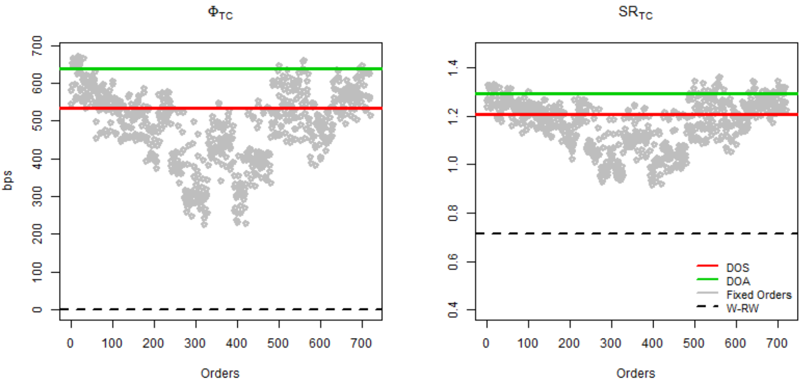

Figure: Economic performance relative to the Wishart-Random-Walk model.
i) Left panel: Annualized Managment Fees (Φ);
ii) Right panel: Sharp Ratios.
All results are already net of transaction costs (TC = 10 bps).

Best Fixed Orders in 2006 and Out-of-Sample for 07-2016

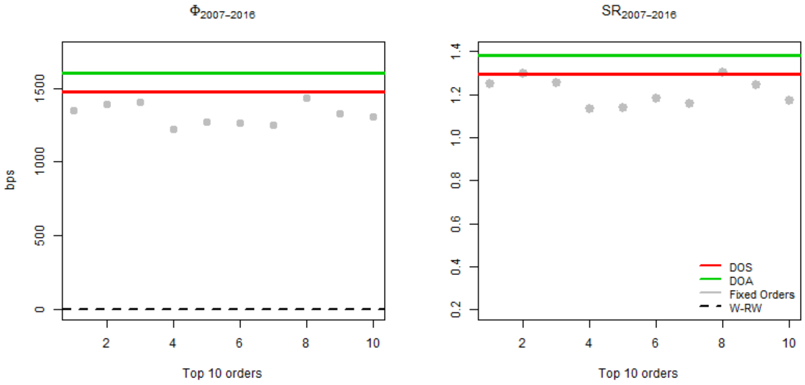

Figure: Economic Performance 2007-2016: DOA and DOS against top 10 orders at the end of 2006.

Macroeconomic Forecasting

Vector Autoregressive (VAR) models are commonly applied in the macroeconomic literature and used in Central Banks and financial institutions in many different contexts.

VARs are known to be a powerful tool to predict the future movements of the economy and for monetary policy evaluation (Sims, 1980, Litterman, 1986, Primiceri, 2005, Clark and McCracken, 2010 and Koop and Korobilis, 2013, Kastner and Huber, 2020).

Inspired by the Cholesky-style behind the work of Primiceri (2005) and Del Negro and Primiceri (2015), we are motivated to explore the ability of our approach to deal with the problem of order uncertainty in a macroeconomic context.

Macroeconomic Forecasting

Since the macroeconomy is continuously adapting to new environments and different sources of breaks, such as wars, global crisis and pandemics, VAR models are strongly susceptible to instabilities.

Those instabilities can induce different sources of dependencies among economic variables, dynamically changing from year to year or just in few months.

The out-of-sample forecasting results can be seriously harmed when considering a static behavior of economic series dependencies.

Macroeconomic Forecasting

Similar to Zhao et al. (2016), now the predictors will be composed by the time series lagged values, building on the format of VARs with time-varying parameters and stochastic volatilities (TVP-VAR-SV).

As Primiceri (2005), we focus on a VAR model with three important US macroeconomic variables: **inflation, unemployment and interest rates.**

Quarterly data for the US economy from 1953Q1 to 2015Q2. Data from Federal Reserve Bank of Philadelphia and St. Louis (also easily available from the R package **bvarsv** (Krueger, 2015)).

Choose between constant or time-varying parameters: δ and $\kappa \in \{0.95, 0.99, 1\}$.

Benchmark Model: The same fixed order used in Primiceri (2005) ($y = [Inf., Unemp., Int.]$)

Statistical Evaluation

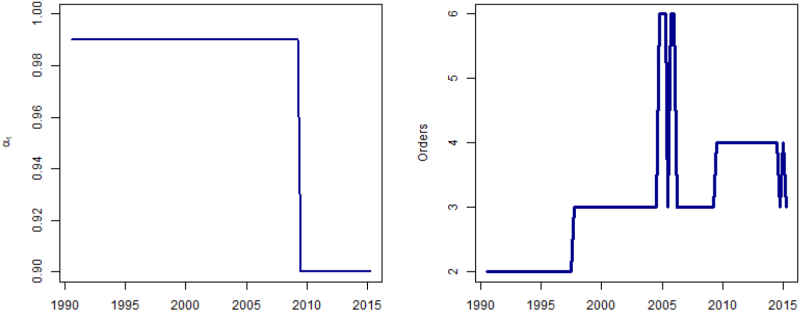

Figure: Time-Varying Forgetting Factor α_t (Left panel) and Order Selection (Right panel)

Statistical Evaluation

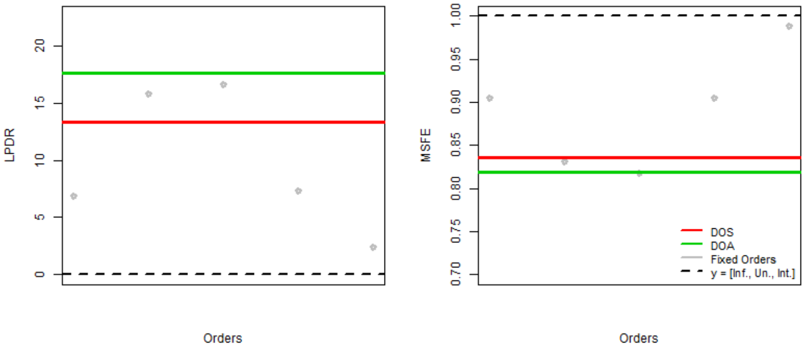

Figure: Relative statistical performance relative to the benchmark. i) Left panel: Log Predictive Density Ratio (LPDR); ii) Right panel: Mean Square Forecast Error (MSFE)

Statistical Evaluation

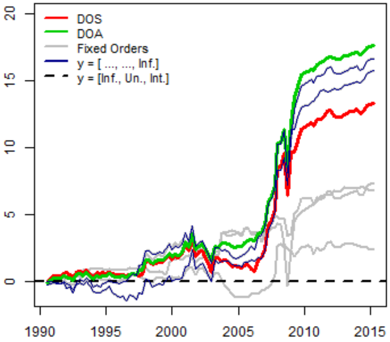

Figure: Accumulated Log Predictive Likelihood relative to the benchmark.

Conclusion

- We introduce a flexible approach to model and forecast multivariate series in the case of uncertainty around the contemporaneous relations among dependent variables.
- The econometrician is able to sequentially learn the dynamic importance of orders when faced with the Cholesky-style model and produce forecasts in decion-making problems.
- We perform two econometric applications to show the importance of ordering learning over time.

Conclusion

- In a dynamic asset allocation we show that the DOL approach generated significant statistical and economic improvements compared to models with fixed orders over time and with the traditional Wishart-Random Walk.
- A mean-variance investor will be willing to pay a considerable managment fee to switch from the traditional Wishart-Random Walk model to the DOL method.
- In a macroeconomic forecasting problem, we show that the econometrician is able to adapt to new economic environments, learning from past mistakes and updating believes about different economic contemporaneous dependencies over time.
- The DOL approach substantially increases point and density forecast accuracy compared to a standard order structure commonly used in the macroeconomic literature.

Conclusion

Thank you!

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Additional Results - Portfolio Allocation

Table: Statistical Performance Relative to DOA

	MSFE	LPDR
DOA-CP-CV	1.01	−103.73
DOA-TVP-CV	1.01	−66.99
DOA-CP-SV	1.01	−12.83
DOS-CP-CV	1.02	−115.77
DOS-TVP-CV	1.01	−81.97
DOS-CP-SV	1.01	−33.02
W-RW-CV	1.07	−159.62
W-RW-SV	1.07	−70.87

Additional Results - Portfolio Allocation

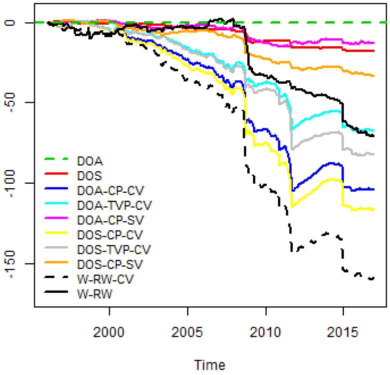

Figure: Accumulated Log Predictive Likelihood relative to the DOA approach.

Additional Results - Portfolio Allocation

Table: Potfolio Performance

	SR	Φ
DOA-CP-CV	1.32	724.44
DOA-TVP-CV	1.18	616.13
DOA-CP-SV	1.33	625.94
DOS-CP-CV	1.26	615.16
DOS-TVP-CV	1.16	663.34
DOS-CP-SV	1.25	600.59
W-RW-CV	0.56	-179.14
W-RW-SV	0.72	0.00

Additional Results - Macroeconomic Forecasting

Table: Statistical Performance Relative to DOA

	MSFE	LPDR
DOA-CP-CV	1.16	−51.11
DOA-TVP-CV	1.05	−42.95
DOA-CP-SV	1.08	−12.66
DOS-CP-CV	1.13	−49.52
DOS-TVP-CV	1.03	−41.33
DOS-CP-SV	1.09	−18.34

Additional Results - Macroeconomic Forecasting

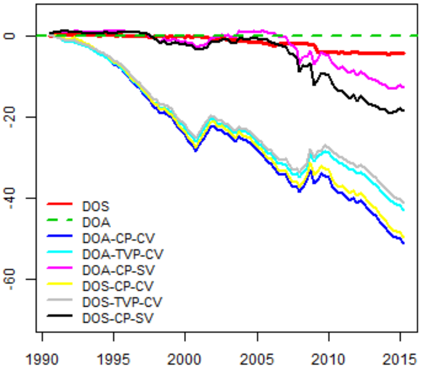

Figure: Accumulated Log Predictive Likelihood

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