Third homework assignment

PhD in Business Economics | Course: Econometrics III
Professor: Hedibert Freitas Lopes | Due date: 10h30, March 12th, 2020.

You can work individually or in pairs.

Use, preferably, `Rmarkdown` (via `RStudio`) to produce your report in PDF or HTML.

Fitting an SV-AR(1) model

An stochastic volatility-AR(1) model for (log)-returns y_t ($t = 1, \ldots, n$) is a state-space model (dynamic model) that assumes that

$$y_t | \sigma_t^2 \sim N(0, \sigma_t^2), \qquad t = 1, \ldots, n,$$

while

$$\log \sigma_t^2 = h_t = \beta_0 + \beta_1 h_{t-1} + \varepsilon_t, \qquad \varepsilon_t \sim N(0, \tau^2),$$

with $0 < \beta_1 < 1$ to ensure stationarity of the log-volatility process[1]. Use my very old `R` code to simulate some synthetic data:

```
set.seed(123456789)
beta0    = -0.00645
beta1    = 0.99
tau2     = 0.15^2
h0       = 0.0
n        = 1000
h        = rep(0,n)
error    = rnorm(n,0.0,sqrt(tau2))
h[1] = beta0+beta1*h0+error[1]
for (t in 2:n)
  h[t] = beta0+beta1*h[t-1]+error[t]
sig2 = exp(h)
y = rnorm(n,0,sqrt(sig2))
plot(y,type="l")
```

For real data, use the return of Petrobrás and Google for the last 1000 business days. Fit a SV-AR(1) model for the three time-series using my code above. Compare the results (and the speed!) of the `R` package `stochvol` of Gregor Kastner. Notice that my prior set-up might be slightly different from that implemented in Gregor's package.

[1] An alternative parametrization includes the log-volatility mean, i.e. $h_t = \mu + \phi(h_{t-1} - \mu) + \varepsilon_t$, where $\varepsilon_t \sim N(0, \omega^2)$.