Take home exam

PhD in Business Economics

Course: Bayesian Econometrics

Professor: Hedibert Freitas Lopes

Start: 6am, February 22nd, 2019.

End: 6pm, February 24th, 2019.

Simple hierarchical regression on time: Souza (1999)[1] considers a number o f hierarchical and dynamic models to describe the nutritional pattern o f pregnant women (data appear at the end of this document). The data depicted in Figure 5.5 (page 161) of Gamerman and Lopes (2006) consist of the weight gains (y_{ij}s) of $I = 68$ pregnant women at 5 to 7 visits (x_{ij}s) to the Instituto de Puericultura e Pediatria Martagão Gesteira from the Universidade Federal do Rio de Janeiro. One of the simplest models she adopted was the simple hierarchical regression on time where

$$\begin{aligned} y_{ij}|\alpha_i, \beta_i, \sigma^2 &\sim N(\alpha_i + \beta_i x_{ij}, \sigma_y^2) \\ (\alpha_i, \beta_i)'|\alpha, \beta &\sim N((\alpha, \beta)', \text{diag}(\sigma_\alpha^2, \sigma_\beta^2)) \\ (\alpha, \beta) &\sim N(0_2, 1000 I_2), \end{aligned}$$

for $i = 1, \ldots, I = 68$, $n = n_1 + \cdots n_I = 427$. The variance parameters $\sigma_y^2, \sigma_\alpha^2$ and σ_β^2 are all *a priori* independent $IG(0.001, 0.001)$.

(a) Fit, by OLS, the 68 individual models for comparison with (c) and (d).

(b) Fit, by OLS, the pooled model, where $(\alpha_i, \beta_i) = (\alpha, \beta)$ for all I, also for comparison.

(b) Show that all full conditionals of the above simple hierarchical regression on time are of known form. More precisely, they are all either Gaussian or Inverse-Gamma densities, therefore suitable for a standard Gibbs Sampler scheme.

(d) Obtain posterior inference for the above simple hierarchical regression on time. Compare the results with those found in (a) and (b).

[1] Aparecida D. P. Souza (1999) Approximate Methods in Bayesian Dynamic Hierarchical Models, unpublished Ph.D. Thesis, COPPE-UFRJ (in Portuguese).

```
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             0.2,  1.4,  3.6,  5.8,  8.3,  9.0,  NA,
             3.8,  3.4,  5.4,  8.8, 10.4, 12.3, 15.3,
             1.9,  5.9,  6.0, 10.6, 14.0, 21.1,  NA,
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             2.0,  2.2,  3.7,  6.5,  9.0,  8.5,  NA,
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          -2.6, -3.2, -2.0,  1.0,  3.0,  4.4,  NA),7,68)


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     11.286, 14.857, 21.286, 25.000, 30.000, 34.429, 39.000,
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group <- c(1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,1,2,2,2,2,2,2,2,2,
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