2nd homework assignment

PhD in Business Economics | Course: Econometrics III
Professor: Hedibert Freitas Lopes | Due date: February 19th, 2019.

You can work individually or in pairs.

The time series in the file `data-hw2.txt` contains 200 observations.

A. Fit an AR(2) to the data via maximum likelihood estimation. For $t = 1, \ldots, n$,

$$y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + \varepsilon_t \qquad \varepsilon_t \;\; iid \;\; N(0, \sigma^2).$$

Assume that y_1 and y_2 are initial values and use the remainder $n = 198$ observations for estimation of $\theta = (\alpha_1, \alpha_2, \sigma^2)$. Explain your findings.

B. Fit an AR(2) to the data from a Bayesian viewpoint. Assume that $p(\theta) = p(\alpha_1|\sigma^2)p(\alpha_2|\sigma^2)p(\sigma^2)$. More precisely, assume that α_1 and α_2 are, *a priori*, $N(0.5, 1.0\sigma^2)$, and that σ^2 is $IG(3, 3)$. The posterior of θ follows a normal-inverse gamma distribution, i.e. $(\alpha_1, \alpha_2|\sigma^2, y^n)$ is bivariate normal and $(\sigma^2|y^n)$ is inverse gamma, where $y^n = (y_1, \ldots, y_n)$. The marginal posterior of (α_1, α_2) is Student's t. Show all these derivations and compare the results with the MLE from part A.

C. Fit an AR(1) plus noise to the data from a Bayesian viewpoint. The AR(1) plus noise can be written as

$$\begin{aligned} y_t &= x_t + v_t & v_t \;\; iid \;\; N(0, V) \\ x_t &= \beta x_{t-1} + w_t & w_t \;\; iid \;\; N(0, W), \end{aligned}$$

with $x_0 \sim N(0, 1)$. Notice that now $\theta = (\beta, W, V)$ and $x^n = (x_1, \ldots, x_n)$ are both unknown. Assume independent priors for the components of θ. More precisely, assume that β is, *a priori*, $N(0.5, 1.0)$, and that W and V are $IG(3, 3)$. Posterior inference is approximated by the Gibbs sampler. More precisely, the joint posterior, $p(\beta, V, W, x^n|y^n)$, can be sampled from by cycling through the full conditionals:

$$\begin{aligned} \text{Gaussian} &: \; p(\beta|V, W, x^n, y^n) \\ \text{Inverse Gamma} &: \; p(V|\beta, W, x^n, y^n) \\ \text{Inverse Gamma} &: \; p(W|\beta, V, x^n, y^n) \\ \text{FFBS} &: \; p(x^n|\beta, V, W, y^n), \end{aligned}$$

where FFBS stands for *forward filtering, backward sampling*, the MCMC algorithm that jointly samples states in Gaussian and linear state space models. Compare your findings with those of part A and part B.