ERRATA FOR DENOMINATOR

$$p(\underline{x}|\quad) = \int_0^\infty \left[\prod_{i=1}^{n} (2\pi\sigma^2)^{-\frac{1}{2}} e^{-\frac{x_i^2}{2\sigma^2}}\right] \frac{(\nu_0\sigma_0^2/2)^{\nu_0/2}}{\Gamma(\frac{\nu_0}{2})} \sigma^{2-(\frac{\nu_0}{2}+1)} e^{-\frac{\nu_0\sigma_0^2/2}{\sigma^2}} d\sigma^2$$

$$= K\int_0^\infty (\sigma^2)^{-\frac{n}{2}} e^{-\frac{\sum x_i^2/2}{\sigma^2}} (\sigma^2)^{-(\frac{\nu_0}{2}+1)} e^{-\frac{\nu_0\sigma_0^2/2}{\sigma^2}} d\sigma^2$$

where $K = \dfrac{(2\pi)^{-\frac{n}{2}}(\nu_0\sigma_0^2/2)^{\nu_0/2}}{\Gamma(\nu_0/2)}$

$$\Rightarrow p(x) = K\int_0^\infty \underbrace{(\sigma^2)^{-(\frac{\nu_0+n}{2}+1)} e^{-\frac{(\nu_0\sigma_0^2+\sum x_i^2)/2}{\sigma^2}}}_{\text{Kernel of } IG\left(\frac{\nu_0+n}{2}, \frac{\nu_0\sigma_0^2+\sum x_i^2}{2}\right)} d\sigma^2$$

$$= K\,\Gamma\left(\frac{\nu_0+n}{2}\right)\cdot\left(\frac{\nu_0\sigma_0^2}{2} + \frac{\sum x_i^2}{2}\right)^{-\left(\frac{\nu_0+n}{2}\right)}$$

$$= K_1\left(1 + \frac{\sum x_i^2}{2}\cdot\frac{2}{\nu_0\sigma_0^2}\right)^{-\left(\frac{\nu_0+n}{2}\right)} \qquad K_1 = K\,\Gamma\left(\frac{\nu_0+n}{2}\right)\Big/\frac{\nu_0\sigma_0^2}{2}$$

$$= K_1\left(1 + \frac{1}{\nu_0}\cdot\sum x_i^2/\sigma_0^2\right)^{-\left(\frac{\nu_0+n}{2}\right)} \Rightarrow X \sim t_{\nu_0}(\underline{0}, \sigma_0^2 I_n)$$

MULTIVARIATE