MIDTERM EXAM

PhD in Business Economics
Course: Econometrics III
Instructor: Hedibert Freitas Lopes
Due date (take home part): 10:30am, May 18th, 2017.

Problem 1 (6.0): Suppose that the observed time series $y_1, \ldots, y_n$ is the result of the sum of two independent and Gaussian (unobservable) time series processes: a driftless AR(1) process and a simple white noise process. More precisely, for $t = 1, \ldots, n$, $y_t = x_t + \epsilon_t$ where $x_t = \phi x_{t-1} + \omega_t$, $\epsilon_t \sim N(0,1)$, $\omega_t \sim N(0,1)$, $E(\epsilon_t \omega_{t'}) = 0$ for all t, t', and, for simplicity, $x_0 = 0$. This rather simple process is commonly referred to as an *AR(1) plus noise* process.

a)(1.0) Derive the theoretical autocorrelation function of the $\{y_t\}$ process.

b)(2.0) Compute $\hat{y}_{n+h} = E(y_{n+1}|y_1, \ldots, y_n, \phi)$ for $h = 1, 2$, i.e. the 1- and 2-step-ahead forecasts. What are the associated standard deviations of the forecast errors, i.e. the squared root of $E[(y_{n+h} - \hat{y}_{n+h})^2|y_1, \ldots, y_n, \phi)$, for $h = 1, 2$?

c)(3.0) Describe how one would perform maximum likelihood estimation of ϕ. Notice that one can only observe $y_1, \ldots, y_n$, i.e. $x_1, \ldots, x_n$ are unknown (they are parameters!) and should not appear in your likelihood for ϕ (technically, they should be integrated out).

Problem 2 (2.0): Consider the time series $\{a_t, t = 0, \pm 1, \pm 2, \ldots\}$ generated as follows:

$$a_t = \begin{cases} u_t & t \text{ is an even number} \\ \frac{u_{t-1}^2 - 1}{\sqrt{2}} & t \text{ is an odd number} \end{cases},$$

where $\{u_t\}$ is a Gaussian white noise with variance one. Is $\{a_t\}$ a weakly stationary process?

Problem 3 (2.0) (take home) The file `cement.txt`, readily available on the course webpage, contains an index (average 2012=100) of the monthly Brazilian production of cement from January 2002 to February 2017 ($n = 182$ observations). The data was collected from IBGE's SIDRA system (`https://sidra.ibge.gov.br/tabela/3650`). Fit your *best* time series model to the data with the goal of producing point and interval forecasts for the months of March, April and May of 2017. Possible models include AR, MA, ARMA, ARIMA models and seasonal extensions of these models. Report your model search strategy and findings in details along with your `R` code.