HOMEWORK ASSIGNMENT 2

PhD in Business Economics — Course: Econometrics III
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The file `hw2.txt` contains $n = 200$ observations: $y_1, \ldots, y_n$ (1st column) and $x_1, \ldots, x_n$ (2nd column). Fit (and compare) the following three models:

Model 1: LOCAL LEVEL MODEL

$$\begin{aligned} y_t &= \mu_t + \epsilon_t & \epsilon_t \text{ iid } N(0, V) \\ \mu_t &= \mu_{t-1} + w_t & w_t \text{ iid } N(0, W), \end{aligned}$$

where

- $\mu_0 \sim N(2, 9)$,
- $E(\epsilon_t w_s) = 0$ for all t, s,
- The variances V and W are unknown.

Model 2: LINEAR GROWTH MODEL

$$\begin{aligned} y_t &= \mu_t + \epsilon_t & \epsilon_t \text{ iid } N(0, V) \\ \mu_t &= \mu_{t-1} + \delta_{t-1} + w_{1t} & w_{1t} \text{ iid } N(0, W_1) \\ \delta_t &= \delta_{t-1} + w_{2t} & w_{2t} \text{ iid } N(0, W_2), \end{aligned}$$

where

- $\mu_0 \sim N(2, 9)$ and $\delta_0 \sim N(0, 9)$,
- $E(\epsilon_t w_{1s}) = E(\epsilon_t w_{2s}) = E(w_{1t} w_{2s}) = 0$ for all t, s,
- The variances V, W_1 and W_2 are unknown.

Model 3: DYNAMIC REGRESSION MODEL

$$\begin{aligned} y_t &= \alpha_t + \beta_t x_t + \epsilon_t & \epsilon_t \text{ iid } N(0, V) \\ \alpha_t &= \alpha_{t-1} + w_{1t} & w_{1t} \text{ iid } N(0, W_1) \\ \beta_t &= \beta_{t-1} + w_{2t} & w_{2t} \text{ iid } N(0, W_2), \end{aligned}$$

where

- $\alpha_0 \sim N(0, 9)$ and $\beta_0 \sim N(0, 9)$,
- $E(\epsilon_t w_{1s}) = E(\epsilon_t w_{2s}) = E(w_{1t} w_{2s}) = 0$ for all t, s,
- The variances V, W_1 and W_2 are unknown.

Note 1: These models are very simple, so I want you to write your own code to learn about the states and parameters of the models. If might benefit from writing your code with the general linear and Gaussian state space model in mind:

$$\begin{aligned} y_t &= F_t'\theta_t + v_t \qquad && v_t \text{ iid } N(0,V) \\ \theta_t &= G_t\theta_{t-1} + w_t \qquad && w_t \text{ iid } N(0,W), \end{aligned}$$

where F_t is a vector of regressors of dimensional p (equal to 1, 2 and 2 for models 1, 2 and 3 above) and θ_t is also a p-dimensional vector of state variables (μ_t in model 1, μ_t and δ_t in model 2, and α_t and β_t in model 3). The variances V and W and the transition matrix G_t follow directly.

Note 2: If you decide to follow the (not so cool!) old school of frequentism and estimate $\theta = (V, W)$ by maximum likelihood, just remember that in all these models $p(y_1, \ldots, y_n|\theta)$ can be computed as $\prod_{t=1}^n p(y_t|y^{t-1}, \theta)$, where $p(y_t|y^{t-1}, \theta)$ is readily available through the Kalman filter recursions.

Note 3: If you decide to follow the path of the righteous (just kidding!) and proceed as a Bayesian, you may decide to use independent inverse gamma priors for V and W and embed the whole learning scheme (learning *a posteriori* both $x_1, \ldots, x_n$ and θ) into a (standard) Gibbs sampler, i.e. sampling recursively from the full conditional distributions: $p(x_1, \ldots, x_n|V, W, y^n)$, $p(V|x_1, \ldots, x_n, W, y^n)$ and $p(W|x_1, \ldots, x_n, V, y^n)$. Please, read/study the notes, textbook chapters, review articles to master the subject. You might wanna play around with some of the many `R` scripts I have made available through the course webpage.