HOMEWORK ASSIGNMENT 1

PhD in Business Economics — Course: Econometrics III
Professor: Hedibert Freitas Lopes
Teaching assistant: Paloma Vaissman Uribe — Due date: May 11th, 2017.

Problem 1: Suppose that the daily log return of a security follows the model

$$r_t = 0.01 + 0.2r_{t-2} + a_t,$$

where $\{a_t\}$ is a Gaussian white noise series with mean zero and variance 0.02. What are the mean and variance of the return series r_t? Compute the lag-1 and lag-2 autocorrelations of r_t . Assume that $r_{100} = -0.01$, and $r_{99} = 0.02$. Compute the 1- and 2-step-ahead forecasts of the return series at the forecast origin $t = 100$. What are the associated standard deviations of the forecast errors?

Problem 2: Consider a standard Gaussian AR(3) process as follows:

$$y_t = \phi_0 + \sum_{i=1}^{3} \phi_i y_{t-i} + \varepsilon_t,$$

for i.i.d. error terms $\varepsilon_t \sim N(0, \sigma^2)$ and $t = 1, \ldots, n$.

a) Derive its theoretical autocorrelation function ρ_l, for $l = 1, 2, \ldots$

b) In class, we have studied an AR(3) with coefficients (estimated by maximum likelihood) given by $\phi_0 = 0.0047$, $\phi_1 = 0.348$, $\phi_2 = 0.179$ and $\phi_3 = -0.142$. Plot the theoretical ACFs (up to lag 30) based on these coefficients.

c) Based on the coefficients from b) and $\sigma = 0.0985$, generate one sample of size n from the AR(3) process and compute/plot the sample ACF (up to lag 30). Compare it with the theoretical one. Consider $n = 10^2, 10^3, 10^4$ and 10^5 observations.

d) For $n = 100$ and $n = 1000$, generate $S = 50$ datasets from the above AR(3) model.

d.1) For each one of the S datasets, compute the ML estimates of the AR(3) and the empirical ACF (up to lag 30).

d.2) Draw box-plots based on the S estimated coefficients and compare to the true values.

d.3) Plot the theoretical ACF agains the S empirical ones.

d.4) Compute AIC/BIC when fitting $AR(1), AR(2), \ldots, AR(10)$ to each one of the S datasets.

Elaborate on your findings.