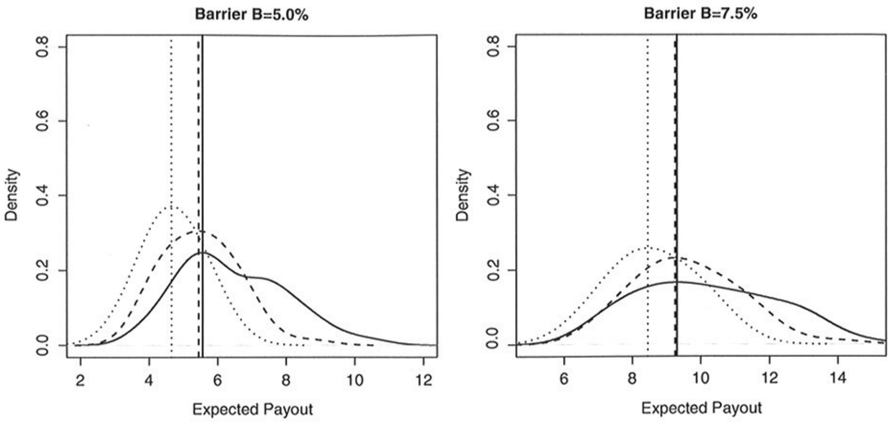

Fig. 34.2 Density estimates of the option payout with density modes. The *solid line* shows the bootstrapped observed data; *the dashed* and *dotted lines* correspond to the estimated vine copula and Student's t copula, respectively

Student's t copula. Our comparison of the payout estimates demonstrates that vine copulas are superior dependence models for financial data.

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Chapter 35
Analysis of Exchange Rates via Multivariate Bayesian Factor Stochastic Volatility Models

Gregor Kastner, Sylvia Frühwirth-Schnatter, and Hedibert F. Lopes

Abstract Multivariate factor stochastic volatility (SV) models are increasingly used for the analysis of multivariate financial and economic time series because they can capture the volatility dynamics by a small number of latent factors. The main advantage of such a model is its parsimony, as the variances and covariances of a time series vector are governed by a low-dimensional common factor with the components following independent SV models. For high-dimensional problems of this kind, Bayesian MCMC estimation is a very efficient estimation method; however, it is associated with a considerable computational burden when the dimensionality of the data is moderate to large. To overcome this, we avoid the usual forward-filtering backward-sampling (FFBS) algorithm by sampling "all without a loop" (AWOL), consider various reparameterizations such as (partial) noncentering, and apply an ancillarity-sufficiency interweaving strategy (ASIS) for boosting MCMC estimation at a univariate level, which can be applied directly to heteroskedasticity estimation for latent variables such as factors. To show the effectiveness of our approach, we apply the model to a vector of daily exchange rate data.

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35.1 Introduction

In the recent years, factor SV models have been progressively applied to important problems in financial econometrics such as asset allocation and asset pricing. These models extend standard factor pricing models such as the arbitrage pricing theory and the capital asset pricing model. As opposed to factor SV models, standard factor pricing models do not attempt to model the dynamics of the volatilities of the asset returns and usually assume that the covariance matrix $\boldsymbol{\Sigma}_t \equiv \boldsymbol{\Sigma}$ is constant. Empirical evidence suggests that multivariate factor SV models are a promising approach for modeling multivariate time-varying volatility, explaining excess asset returns, and generating optimal portfolio strategies. Following [1], the model reads

$$\boldsymbol{y}_t = \boldsymbol{\Lambda} \boldsymbol{f}_t + \boldsymbol{\Sigma}_t^{1/2} \boldsymbol{\epsilon}_t, \qquad \boldsymbol{\epsilon}_t \sim N_m(\boldsymbol{0}, \boldsymbol{I}_m), \tag{1}$$

$$\boldsymbol{f}_t = \boldsymbol{V}_t^{1/2} \boldsymbol{u}_t, \qquad \boldsymbol{u}_t \sim N_r(\boldsymbol{0}, \boldsymbol{I}_r), \tag{2}$$

where for $t = 1, \ldots, T$, the vector $\boldsymbol{y}_t = (y_{1t}, \ldots, y_{mt})'$ consists of (potentially demeaned) log returns of m observed time series, $\boldsymbol{\Sigma}_t = \mathrm{Diag}(\exp(h_{1t}), \ldots, \exp(h_{mt}))$, $\boldsymbol{V}_t = \mathrm{Diag}(\exp(h_{m+1,t}), \ldots, \exp(h_{m+r,t}))$, and $\boldsymbol{\Lambda}$ is an unknown $m \times r$ factor loading matrix with elements Λ_{ij}. The standard assumption is that $\boldsymbol{f}_t$, $\boldsymbol{f}_s$, $\boldsymbol{\epsilon}_t$, and $\boldsymbol{\epsilon}_s$ are pairwise independent for all t and s. Both the latent factors and the idiosyncratic shocks are allowed to follow different stochastic volatility processes, i.e.,

$$h_{it} = \mu_i + \phi_i(h_{i,t-1} - \mu_i) + \sigma_i \eta_{it}, \qquad \eta_{it} \sim N(0, 1). \tag{3}$$

In the following, we identify the model by imposing a lower-triangular structure for $\boldsymbol{\Lambda}$ with unconstrained diagonal elements and therefore set $\mu_i = 0$ for $i \in \{m+1, \ldots, m+r\}$. Clearly, this introduces an order dependence among the responses and makes the appropriate choice of the first r variables an important modeling decision.

35.2 Factor SV Estimation

After fixing $T(m + 2r) + mr + 4m + 3r$, in our application 81763, starting values for (the elements of) $\boldsymbol{\mu}, \boldsymbol{\phi}, \boldsymbol{\sigma}, \boldsymbol{h}, \boldsymbol{f}, \boldsymbol{\Lambda}$, we repeat the following steps:

(a) *Perform $m+r$ univariate SV updates* for $h_{i0}, \ldots, h_{iT}$, ϕ_i, σ_i and m updates for μ_i. We do this by sampling the latent variables AWOL as in [4]; thus, no FFBS methods are required, there is no need to invert the tridiagonal information matrix of the joint conditional distribution of the latent log volatilities and computations are fast due to the availability of band back-substitution already implemented in practically all widely used programming languages. Moreover, we employ several variants of ASIS [5] by moving the parameters of interest from the state equation (3) in its centered parameterization to the augmented observation equation (1) or (2) and perform an extra update for these parameters in the noncentered parameterization. Doing so is very cheap in terms of computation—only around 2 % extra CPU time is needed—and nevertheless has substantial effect on sampling efficiency. The actual sampler is written in C and made accessible through the R package `stochvol` [2], publicly available on CRAN. More details about efficient univariate SV estimation can be found in [3].

(b) *Sample the factor loadings*, constituting m independent r-variate regression problems, from the T-dimensional Gaussian distribution $\boldsymbol{\Lambda}_{i\cdot}|\boldsymbol{f}, \boldsymbol{y}_i, \boldsymbol{h}_i$.

(c) *Sample the latent factors*, constituting T independent r-variate regression problems, from the m-dimensional Gaussian distribution $\boldsymbol{f}_t|\boldsymbol{\Lambda}, \boldsymbol{y}_{\cdot t}, \boldsymbol{h}_{\cdot t}$.

35.3 Application

We apply a three-factor SV model to EUR exchange rates, quoted indirectly. The data stems from the European Bank's Statistical Data Warehouse and comprises $T = 3140$ observations of 20 currencies ranging from January 3, 2000, to April 4, 2012. Figure 35.1 depicts the proportions of the variance which can be

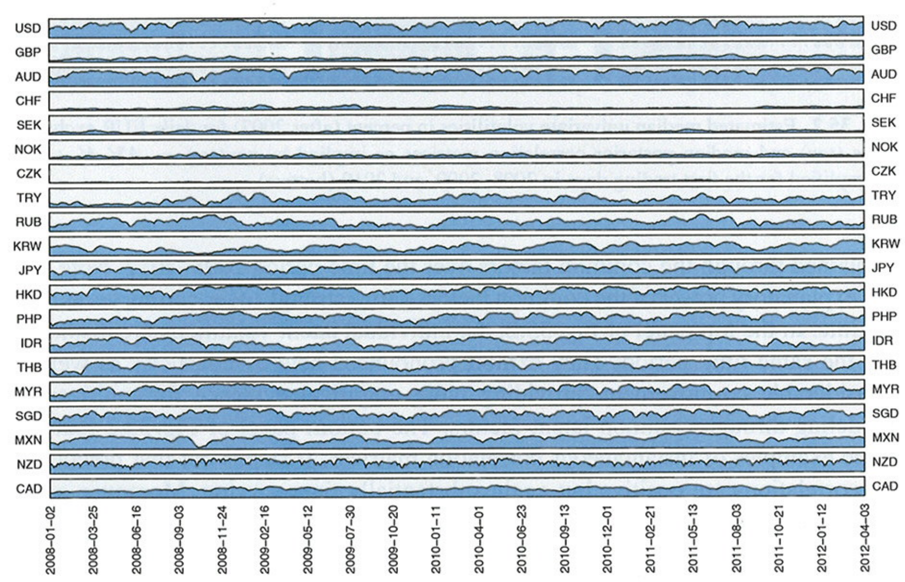

Fig. 35.1 Median posterior proportions of variances explained by the common latent factors given through $1 - \Sigma_{ii,t}/\mathrm{var}(y_{it})$. Results are displayed on a daily basis, for the time from 2008 onwards

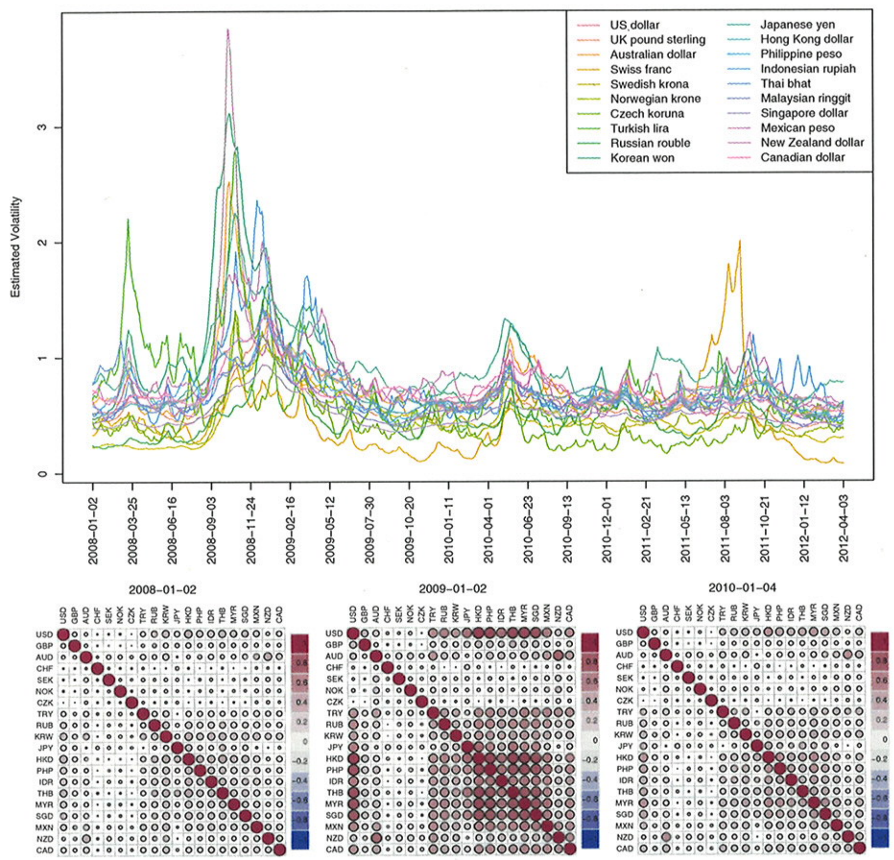

Fig. 35.2 Estimated median univariate volatilities in percent (after 2007) for daily EUR exchange rates (*top*) and median posterior correlation matrices as implied by $\text{cov}(\boldsymbol{y}_t) = \boldsymbol{\Lambda V}_t\boldsymbol{\Lambda}' + \boldsymbol{\Sigma}_t$, exemplified for the first trading days in 2008, 2009, and 2010 (*bottom*)

explained through the common factors over the time period 2008–2012. Note that the explanatory power of the common factors varies strongly over currencies as well as time. In the top panel of Fig. 35.2, the individual latent volatilities are displayed for the same time period. They exhibit pronounced heteroskedasticity as well as considerable co-movement, providing more empirical evidence for multivariate modeling through common latent factors. The bottom panel of Fig. 35.2 features three examples of instantaneous correlation matrices. It stands out that practically all correlations are positive but again substantially time varying. Moreover, some clusters of highly correlated currencies (such as the "Asian Tigers") can be spotted, while continental European currencies show little correlation.

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