Deterministic or Stochastic Trend?

Let us consider two of the simplest versions:

$$\begin{aligned} \text{Deterministic trend (DT)} &: \quad y_t = \beta t + \epsilon_t \\ \text{Stochastic trend (ST)} &: \quad y_t = \beta + y_{t-1} + \epsilon_t, \end{aligned}$$

where ϵ_t is white noise with variance σ^2 ($= 1$, for simplicity) and $y_0 = 0$ (also for simplicity).

It is easy to see that

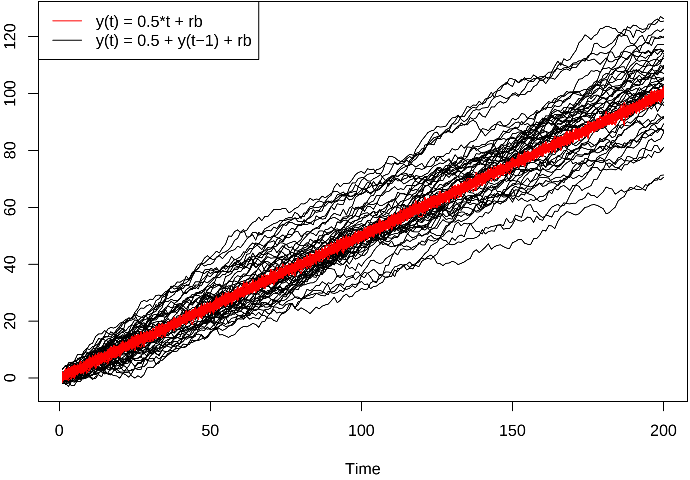
$$E_{DT}(y_t) = E_{ST}(y_t) = \beta t$$

but

$$V_{DT}(y_t) = 1 \quad \text{and} \quad V_{DT}(y_t) = t.$$

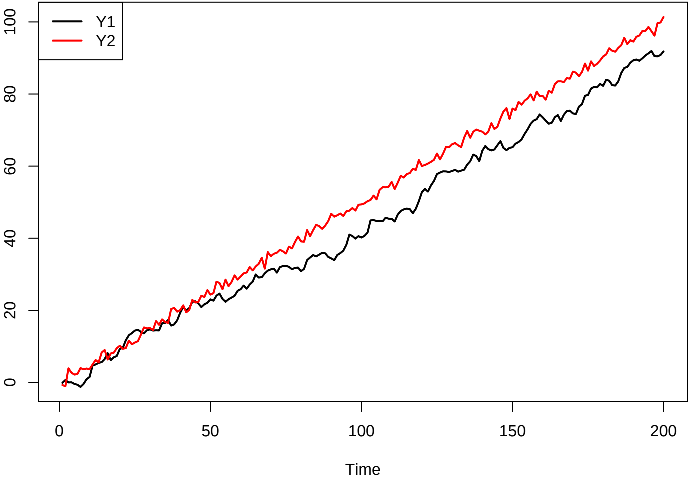
Expectation with respect to all information up to time $t = 0$.

Simulating DT and ST time series



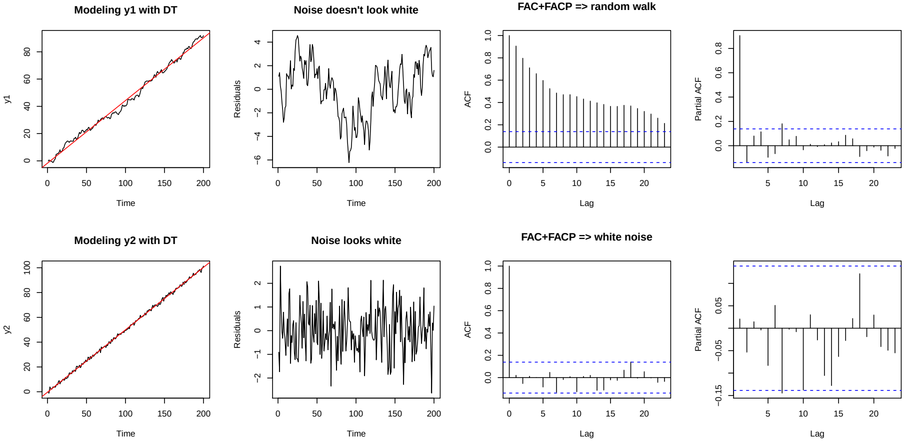
How to model y_{1t} and y_{2t}?

Even with $n = 100$ one can argue that the trend of $\{y_{2t}\}$ "looks" more deterministic than the trend of $\{y_{1t}\}$.



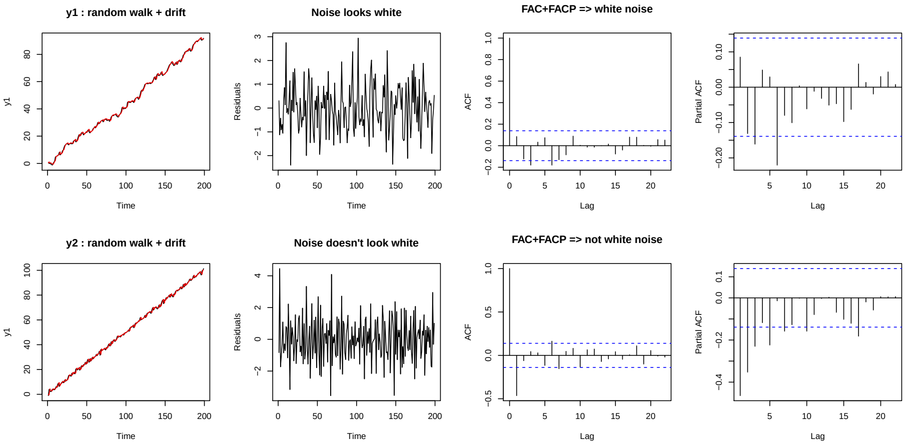
Model y_{1t} and y_{2t} with deterministic trends

Even after removing a determinist trend from y_{1t}, the residuals still behave like a random walk. On the other hand, y_{2t} is definitely *trend-stationary*.



Model y_{1t} and y_{2t} with stochastic trends

After fitting a random walk plus drift for y_{1t}, the residuals behave like a white noise, so y_{1t} is *difference-stationary*.



Fitting a random walk plus drift for y_{2t} (which is trend-stationary), induces an MA(1) behavior in the residuals.

If y_t is trend stationary,

$$y_t = \beta t + \epsilon_t$$

then

$$y_{t-1} = \beta(t-1) + \epsilon_{t-1}$$

and

$$\Delta y_t = \beta + v_t$$

where $v_t = \epsilon_t - \epsilon_{t-1}$, such that $E(v_t) = 0$, $V(v_t) = 2$ and

$$Cov(v_t, v_{t-1}) = Cov(\epsilon_t - \epsilon_{t-1}, \epsilon_{t-1} - \epsilon_{t-2}) = -V(\epsilon_t) = -1$$

and $Cov(v_t, v_{t-h}) = 0$, for $h > 1$. Therefore, the 1st order autocorrelation is

$$\rho(1) = \frac{Cov(v_t, v_{t-1})}{V(v_t)} = -0.5.$$

Summary

If y_t is trend-stationary:

- Stochastic trend fit: residuals with MA(1) behavior.
- Deterministic trend fit: residuals are white noise.

If y_t is difference-stationary:

- Stochastic trend fit: residuals are white noise.
- Deterministic trend fit: residuals are random walk.

Lesson: ALWAYS check the residuals!