

# Simple instrumental variables regressions<sup>1</sup>

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<sup>1</sup>From Wooldridge (2013, Chapter 15)

## Return to education (for women)

Estimating the return (log wages) to education for  $n = 428$  married working women as

$$\text{lwage}_i = \beta_0 + \beta_1 \text{educ}_i + u_i.$$

**OLS:**

$$\widehat{\text{lwage}}_i = \underbrace{-0.1852}_{(0.1852)} + \underbrace{0.1086}_{(0.0144)} \text{educ}_i \quad R^2 = 0.1158.$$

95% C.I. for  $\beta_1$  : (0.0798, 0.1374).

**Conclusion:** Roughly 12% return for another year of education.

# Fathers education as an instrument for education

**1st requirement:**  $cov(\text{fathereduc}, u) = 0$ .

**2nd requirement:**  $cov(\text{fathereduc}, \text{educ}) \neq 0$ .

$$\widehat{\text{educ}}_i = 10.2371 + 0.2694\text{fathereduc}_i \quad R^2 = 0.1706.$$

(0.2759)      (0.0286)

**IV regression:**

$$\widehat{\text{lwage}}_i = 0.441 + 0.059\text{educ}_i \quad R^2 = 0.09.$$

(0.446)      (0.035)

95% C.I. for  $\beta_1$  :  $(-0.011, 0.129)$ .

**Conclusion:** About 6% return to education  $\Rightarrow$  omitted ability bias.

# R output

Call:

```
lm(formula = lwage ~ educ)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	-0.1852	0.1852	-1.000	0.318
educ	0.1086	0.0144	7.545	2.76e-13 ***

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Signif. codes: 0 \*\*\* 0.001 \*\* 0.01 \* 0.05 . 0.1 1

Residual standard error: 0.68 on 426 degrees of freedom  
Multiple R-squared: 0.1179, Adjusted R-squared: 0.1158  
F-statistic: 56.93 on 1 and 426 DF, p-value: 2.761e-13

Call:

```
lm(formula = educ ~ fatheduc)
```

Residuals:

Min	1Q	Median	3Q	Max
-8.4704	-1.1231	-0.1231	0.9546	5.9546

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	10.23705	0.27594	37.099	<2e-16 ***
fatheduc	0.26944	0.02859	9.426	<2e-16 ***

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Signif. codes: 0 \*\*\* 0.001 \*\* 0.01 \* 0.05 . 0.1 1

Residual standard error: 2.081 on 426 degrees of freedom  
Multiple R-squared: 0.1726, Adjusted R-squared: 0.1706  
F-statistic: 88.84 on 1 and 426 DF, p-value: < 2.2e-16

## Return to education (men)

If the number of siblings is an instrument for education, ie.

$$\text{educ}_i = \beta_0 + \beta_1 \text{sibs}_i + u_i,$$

so

$$\widehat{\text{educ}}_i = 14.1388 - 0.2279 \text{sibs}_i \quad R^2 = 0.05625.$$

(0.1131)            (0.0303)

Assuming that  $\text{cov}(\text{sibs}, u) = 0$ , then the IV fit is

$$\widehat{\text{lwage}}_i = 5.13 + 1.122 \text{sibs}_i$$

(0.36)            (0.026)

**OLS:**  $\hat{\beta}_1 = 0.0598$  with a standard error of 0.006 and  $R^2 = 0.096$ .

**Conjecture:** Maybe more siblings means, on average, less parental attention, which could result in lower ability.

# R output

Call:

```
lm(formula = lwage ~ educ)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	5.973062	0.081374	73.40	<2e-16 ***
educ	0.059839	0.005963	10.04	<2e-16 ***

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Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4003 on 933 degrees of freedom

Multiple R-squared: 0.09742, Adjusted R-squared: 0.09645

F-statistic: 100.7 on 1 and 933 DF, p-value: < 2.2e-16

Call:

```
lm(formula = educ ~ sibs)
```

Coefficients:

	Estimate	Std. Error	t value	Pr(> t )
(Intercept)	14.13879	0.11314	124.969	< 2e-16 ***
sibs	-0.22792	0.03028	-7.528	1.22e-13 ***

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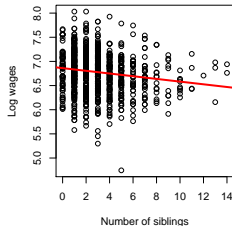
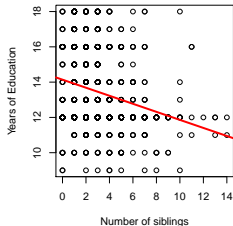
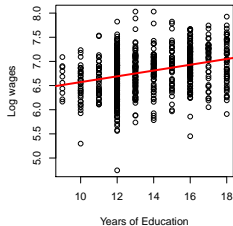
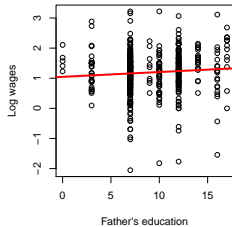
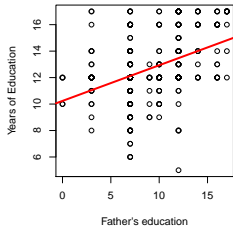
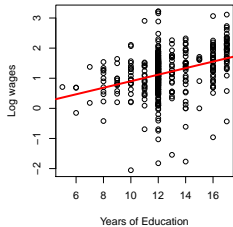
Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.134 on 933 degrees of freedom

Multiple R-squared: 0.05726, Adjusted R-squared: 0.05625

F-statistic: 56.67 on 1 and 933 DF, p-value: 1.215e-13

# Graphical summaries



## Binary/categorical instrument

Angrist and Krueger (1991) proposed **frstqrt** (=1 if born in the 1st quarter of the year) as an instrumental variable for education.

$$\begin{aligned} \text{cov}(\ln\text{wage} - \beta_0 - \beta_1\text{educ}, \text{frstqrt}) &= 0 \\ \text{cov}(\text{ability}, \text{frstqrt}) &= 0 \end{aligned}$$

Compulsory school attendance  $\implies \text{cov}(\text{educ}, \text{frstqrt}) \neq 0$ .

Years of education varies only slightly across quarter of birth.  
Based on  $n = 247,199$  they found that

- ▶ OLS:  $\hat{\beta}_1 = 0.0801$  (standard error 0.0004)
- ▶ IV:  $\hat{\beta}_1 = 0.0715$  (0.0219).

**Headache:** Even a small amount of correlation between  $z$  and  $u$  can cause serious problems for the IV estimator.



# Graphical summaries

