BAYESIAN ECONOMETRICS — HEDIBERT FREITAS LOPES

FINAL EXAM - TAKE HOME — APRIL 11TH 2014

Regression model with an unknown changepoint. Suppose that the density for y_t, $t = 1, 2, \ldots, T$, can be expressed as

$$(y_t|x_t, \theta_1, \theta_2, \beta_1, \beta_2, \sigma^2, \tau^2, \lambda) \sim \begin{cases} N(\theta_1 + \theta_2 x_t, \sigma^2) & t \leq \lambda \\ N(\beta_1 + \beta_2 x_t, \tau^2) & t > \lambda \end{cases},$$

and that you employ priors of the form $(\theta_1, \theta_2)' \sim N(\mu_\theta, V_\theta)$, $(\beta_1, \beta_2)' \sim N(\mu_\beta, V_\beta)$, $\sigma^2 \sim IG(a_1, b_1)$, $\tau^2 \sim IG(a_2, b_2)$, and $\lambda \sim U\{1, 2, \ldots, T-1\}$.

Simulation. Generate $T = 500$ observations from the changepoint model $y_t|x_t \sim N(2 + x_t, 0.2)$ for $t \leq 85$ and $N(1.5 + 0.8x_t, 0.5)$, for $t > 85$, where $x_t \sim N(0, 1)$. Select prior hyperparameters as follows: $\mu_\theta = \mu_\beta = 0_2$, $V_\theta = V_\beta = 100I_2$, $a_1 = b_1 = 3$, and $a_2 = b_2 = 0.5$. sample from the joint posterior, $p(\theta_1, \theta_2, \beta_1, \beta_2, \sigma^2, \tau^2, \lambda|y, x)$, via a Gibbs sampler.

Real data. The data below represents yearly homicide rates, y_t, of young men between 15 and 29 years of age (in 100,000 inhabitants) in the city of São Paulo from 1980 to 2009 ($T = 30$).

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67.01 85.89 82.77 119.73 160.40 161.66 166.90 177.10 161.52 197.68
205.04 210.76 195.83 180.30 198.05 223.14 223.86 220.06 238.41 273.12
267.07 257.76 214.29 227.34 168.58 110.06 83.79 63.47 51.43 56.26
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MLE of the two regressions, when $\hat{\lambda} = 1998$ is assumed to be the change point, are given by $y_t = 90 + 8x_t$ ($\hat{\sigma} = 20$), for $t \leq 1998$ and $y_t = 822 - 26x_t$ ($\hat{\tau} = 21$), for $t > 1998$, where $x_t = t - 1979$. Assuming the same priors as above, compute the posterior means, standard deviations, medians and 95% posterior credibility intervals of θ_1, θ_2, σ, β_1, β_2, τ and λ.