Bayesian Econometrics **Summer Quarter 2013**

Instructor: Hedibert F. Lopes **2nd review session**

Solution

Single-parametric Bayes learning. Let us consider the following simple linear regression application, illustrated using a sample of $n = 1,217$ observations from the National Longitudinal Survey of Youth (NLSY):

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad \epsilon_i \stackrel{iid}{\sim} N(0, \sigma^2),$$

with y_i referring to the log hourly wage received by individual i and x_i the years of schooling completed by individual i. The file `logwages-yearsofeducation.txt` contains the data. Let $\mathcal{D}_n$ be the set $\{y_1, \ldots, y_n, x_1, \ldots, x_n\}$, with $\mathcal{D}_0$ being the set of initial knowledge.

a) Fix $(\beta_0, \sigma^2) = (1.17, 0.267)$ and let $(\beta_1|\mathcal{D}_0) \sim N(0, 2)$. Derive $p(\beta_1|\mathcal{D}_n)$. Draw in one frame the likelihood, the prior and the posterior.

b) Fix $(\beta_1, \sigma^2) = (0.091, 0.267)$ and let $(\beta_0|\mathcal{D}_0) \sim N(0, 2)$. Derive $p(\beta_0|\mathcal{D}_n)$. Draw in one frame the likelihood, the prior and the posterior.

c) Fix $(\beta_0, \beta_1) = (1.17, 0.091)$ and let $(\sigma^2|\mathcal{D}_0) \sim IG(3, 0.4)$. Derive $p(\beta_0|\mathcal{D}_n)$. Draw in one frame the likelihood, the prior and the posterior.

Multi-parametric Bayes learning + Monte Carlo approximation.

d) Fix $\sigma^2 = 0.267$ and let $(\beta_0, \beta_1|\mathcal{D}_0) \sim N(0, 2I_2)$. Derive $p(\beta_0, \beta_1|\mathcal{D}_n)$. Draw contours of the likelihood, the prior and the posterior (each one in a separate frame).

e) Fix $\sigma^2 = 0.267$ and let $(\beta_1, \beta_2|\mathcal{D}_0) \sim t_\nu(\mu, V)$, where $\nu = 4$, $\mu = 0$ and $V = I_2$. Therefore, the posterior is

$$p(\beta_0, \beta_1|\mathcal{D}) \propto \exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2\right\}\left(1 - \frac{1}{\nu}(\beta_0^2 + \beta_1^2)\right)^{-\frac{\nu+2}{2}}.$$

Draw contours of the likelihood function, the prior density and the posterior density (each one in a separate frame).

f) Use *Sampling Importance Resampling (SIR)* to approximate the following quantities from e) above.

f.1) $E(\beta_1|\mathcal{D})$,

f.2) $V(\beta_1|\mathcal{D})$,

f.3) $Pr(\beta_1 > 0.1|\mathcal{D})$,

f.4) $p(\beta_0 + 12\beta_1|\mathcal{D})$.

Try various Monte Carlo sizes, such as $M = 1000$, $M = 10000$ and $M = 100000$ in order to access the size of the Monte Carlo error.

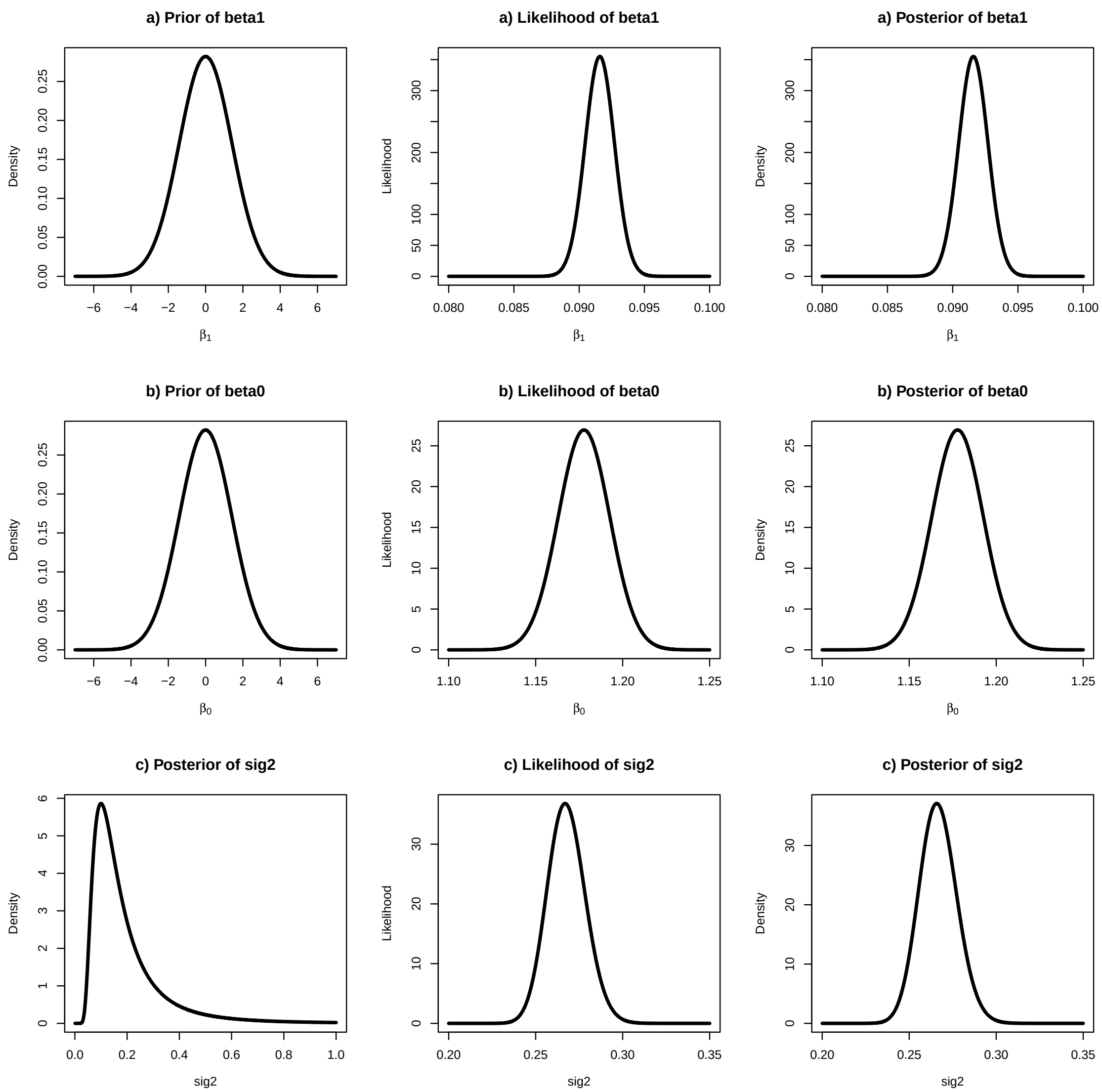

Figure 1: Prior, likelihood and posterior for items a), b) and c).

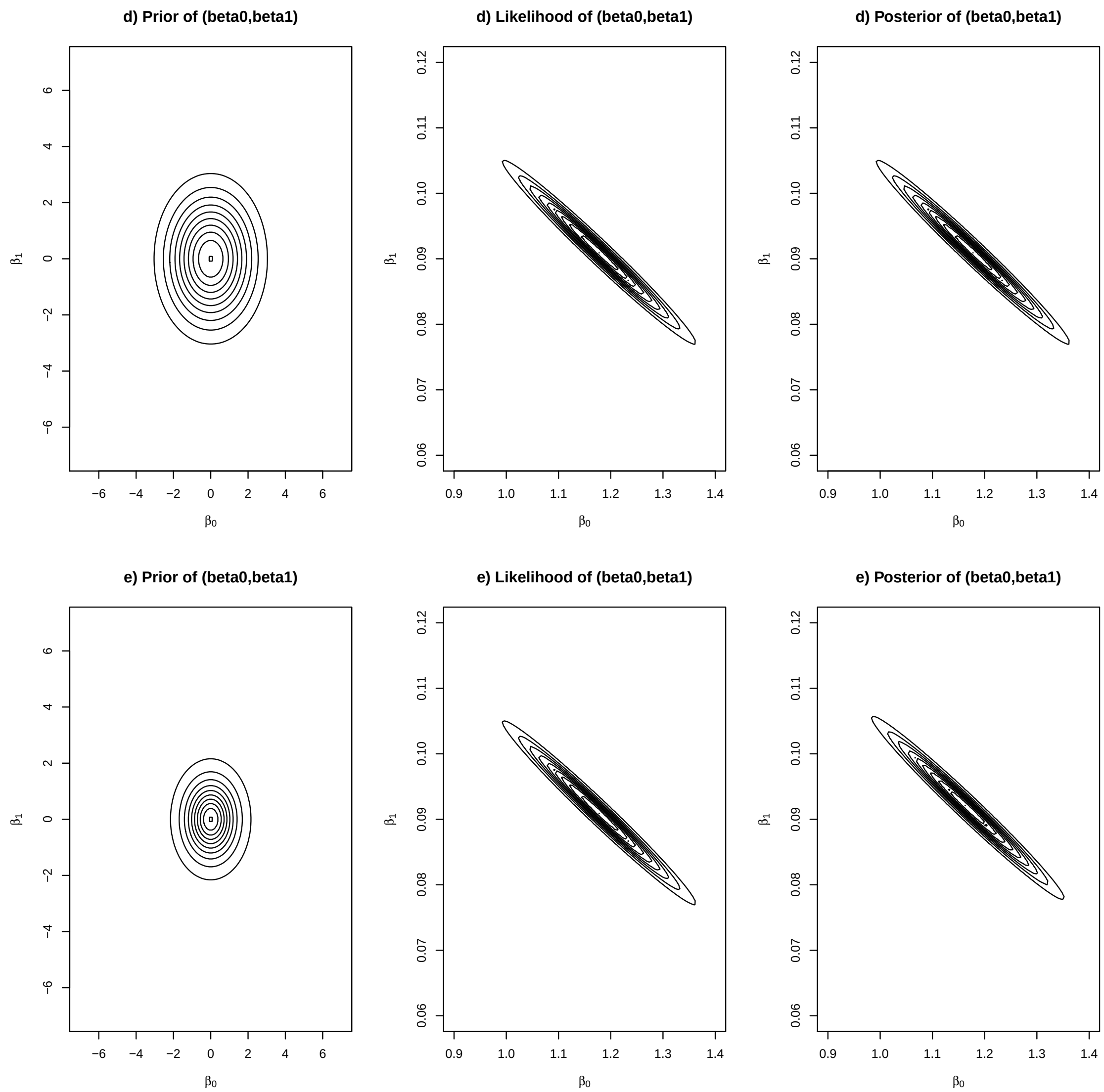

Figure 2: Joint posterior of (β_0, β_1) given Normal prior (top row) and given the Cauchy prior (bottom row).

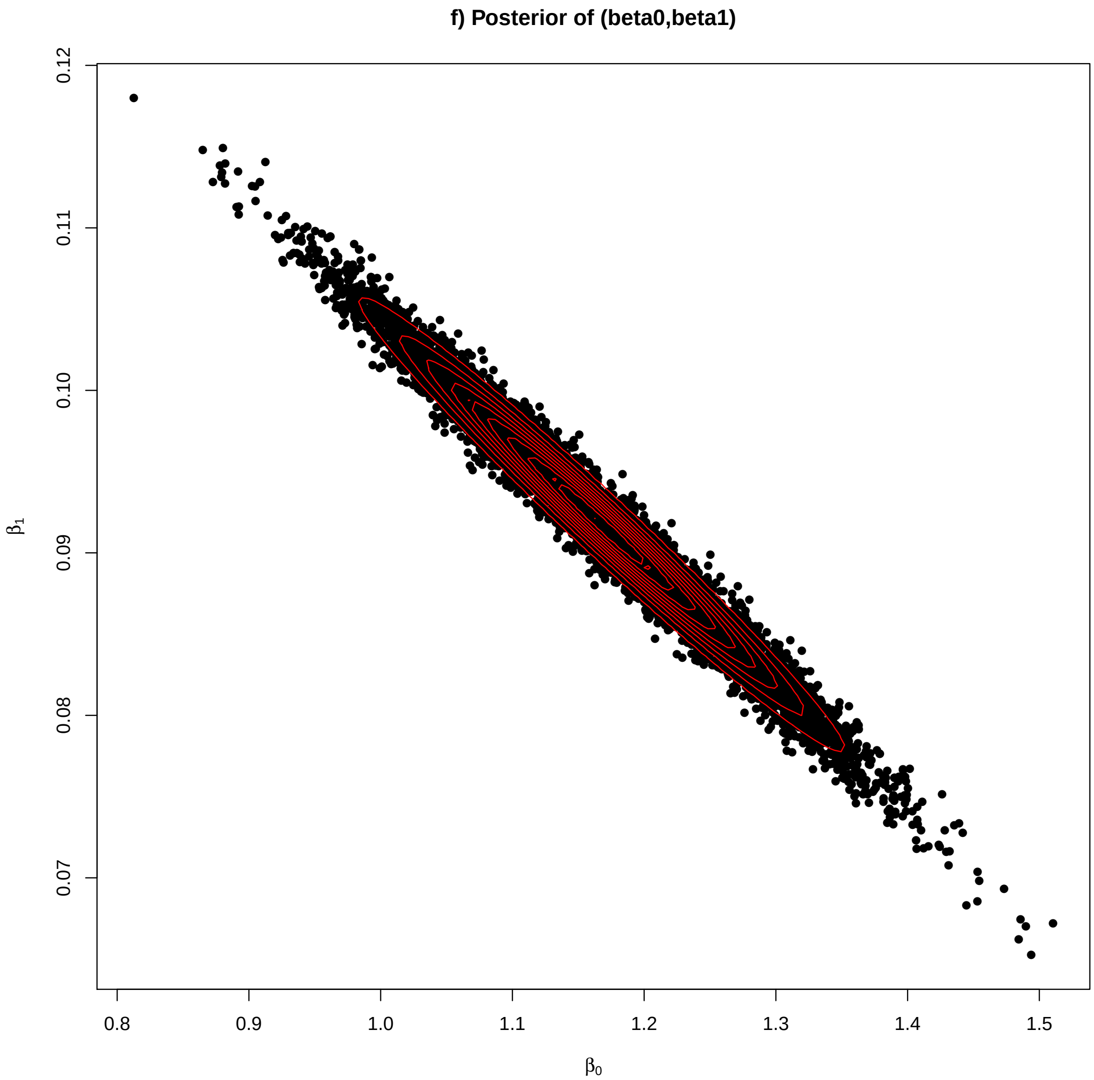

Figure 3: Implementation of a SIR-based algorithm that uses $N(\beta; \hat{\beta}, \sigma^2(X'X)^{-1})$ as proposal/candidate/auxiliary distribution. Posterior draws along with posterior contours.

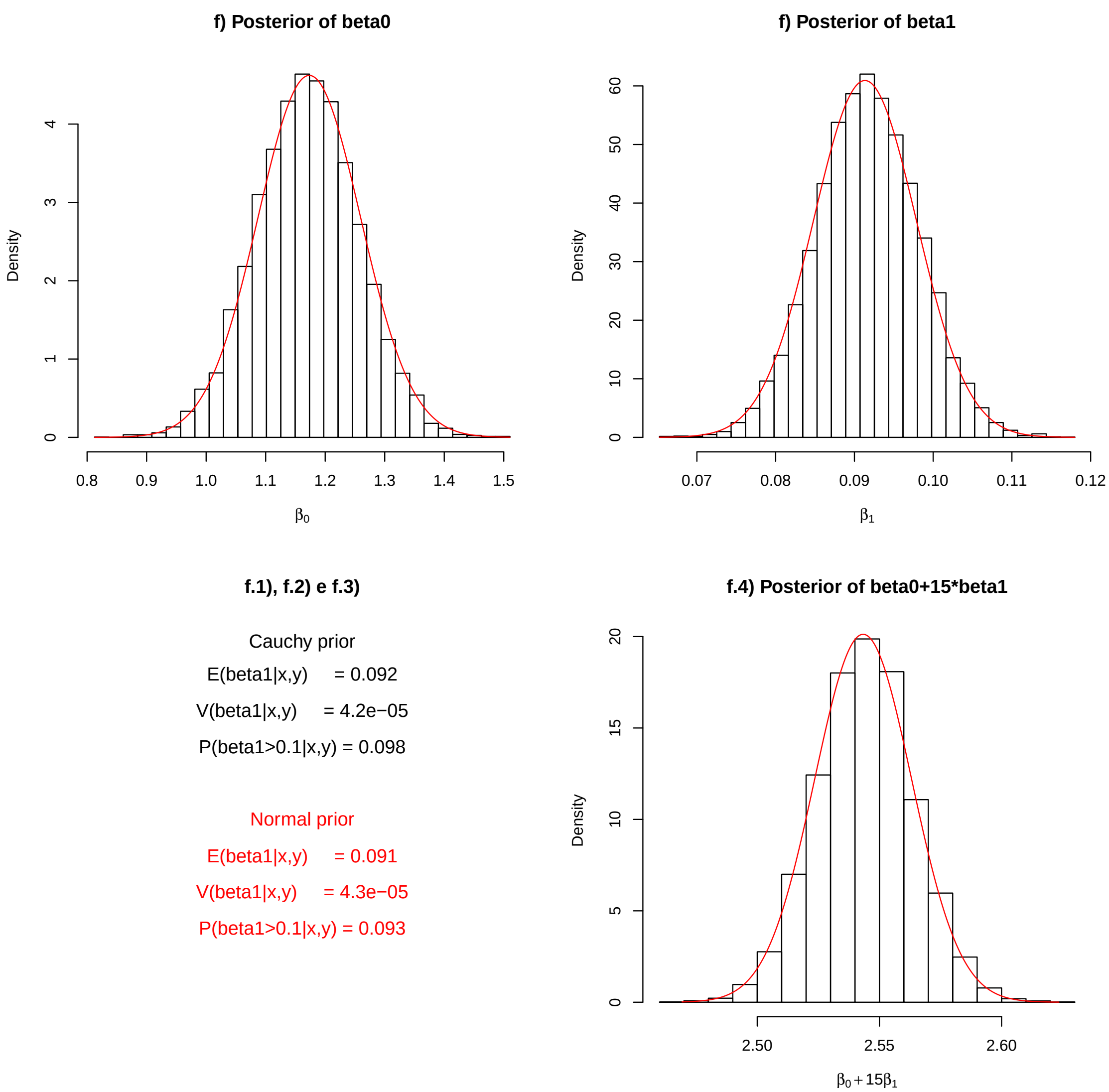

Figure 4: Comparing the posterior distributions $p_N(\beta_0, \beta_1|x, y)$ (based on the normal prior) and $p_C(\beta_0, \beta_1|x, y)$ (based on the Cauchy prior). red : Normal, black: Cauchy.