BAYESIAN PANEL DATA

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I took the following three examples from the WinBugs list of examples. Both WinBugs and the examples can be found at the The BUGS Project - Bayesian inference Using Gibbs Sampling site: http://www.mrcbsu.cam.ac.uk/bugs/welcome.shtml

Example 1: Orange Trees, Non-linear growth curve

This dataset was originally presented by Draper and Smith (1981) and reanalysed by Lindstrom and Bates (1990). The data Yij consist of trunk circumference measurements recorded at time xj, j=1,...,7 for each of i = 1,..., 5 orange trees.

Example 2: London schools

Goldstein et al. (1993) present an analysis of examination results from inner London schools. They use hierarchical or multilevel models to study the between-school variation, and calculate school-level residuals in an attempt to differentiate between `good' and `bad' schools.

Example 3: Repeated measures on Poisson counts

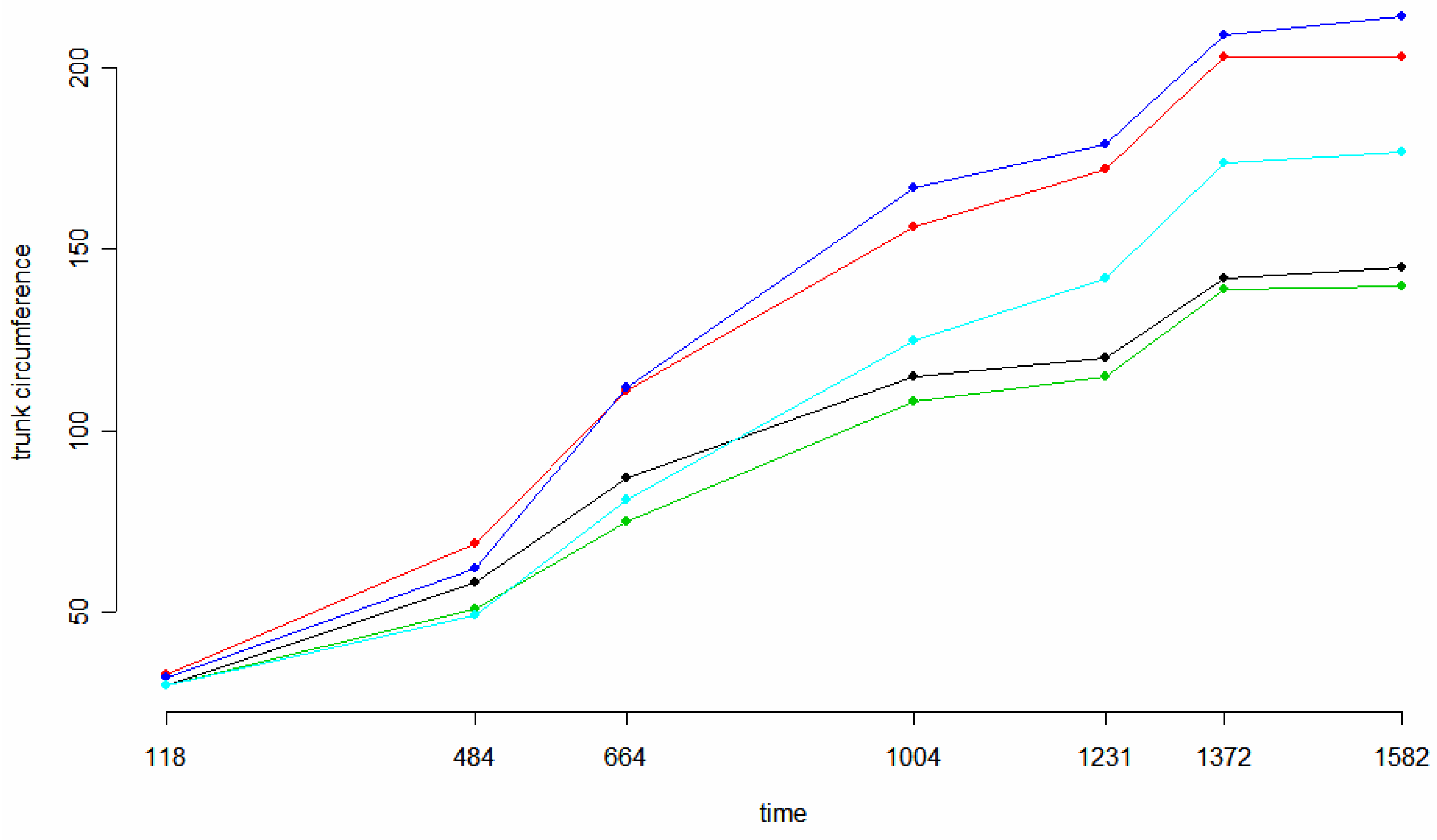
Breslow and Clayton (1993) analyse data initially provided by Thall and Vail (1990) concerning seizure counts in a randomised trial of anti-convulsant threrapy in epilepsy. The table below shows the successive seizure counts for 59 patients. Covariates are treatment (0,1), 8-week baseline seizure counts, and age in years.

Example 1: Non-linear growth curve

This dataset was originally presented by Draper and Smith (1981) and reanalysed by Lindstrom and Bates (1990). The data Yij consist of trunk circumference measurements recorded at time xj, j=1,...,7 for each of i = 1,..., 5 orange trees. We consider a logistic growth curve as follows:

j	xj	y1j	y2j	y3j	y4j	y5j
1	118	30	33	30	32	30
2	484	58	69	51	62	49
3	664	87	111	75	112	81
4	1004	115	156	108	167	125
5	1231	120	172	115	179	142
6	1372	142	203	139	209	174
7	1582	145	203	140	214	177

Time x trunk circumference



Model and prior distribution

$$y_{ij} \sim N\left(\frac{\phi_{i1}}{1+\phi_{i2}e^{\phi_{i3}x_j}}, \tau_C^{-1}\right)$$

where

$$\theta_{i1} = \log(\phi_{i1}), \theta_{i2} = \log(\phi_{i2}+1), \text{ and } \theta_{i3} = \log(-\phi_{i3})$$

$$\begin{pmatrix}\theta_1\\ \theta_2\\ \theta_3\end{pmatrix} \sim N\left[\begin{pmatrix}\mu_1\\ \mu_2\\ \mu_3\end{pmatrix}, \begin{pmatrix}\tau_1^{-1} & 0 & 0\\ 0 & \tau_2^{-1} & 0\\ 0 & 0 & \tau_3^{-1}\end{pmatrix}\right]$$

$$\mu_i \sim N(0,1000)$$

$$\tau_i \sim G(0.001, 0.001)$$

$$\tau_C \sim G(0.001, 0.001)$$

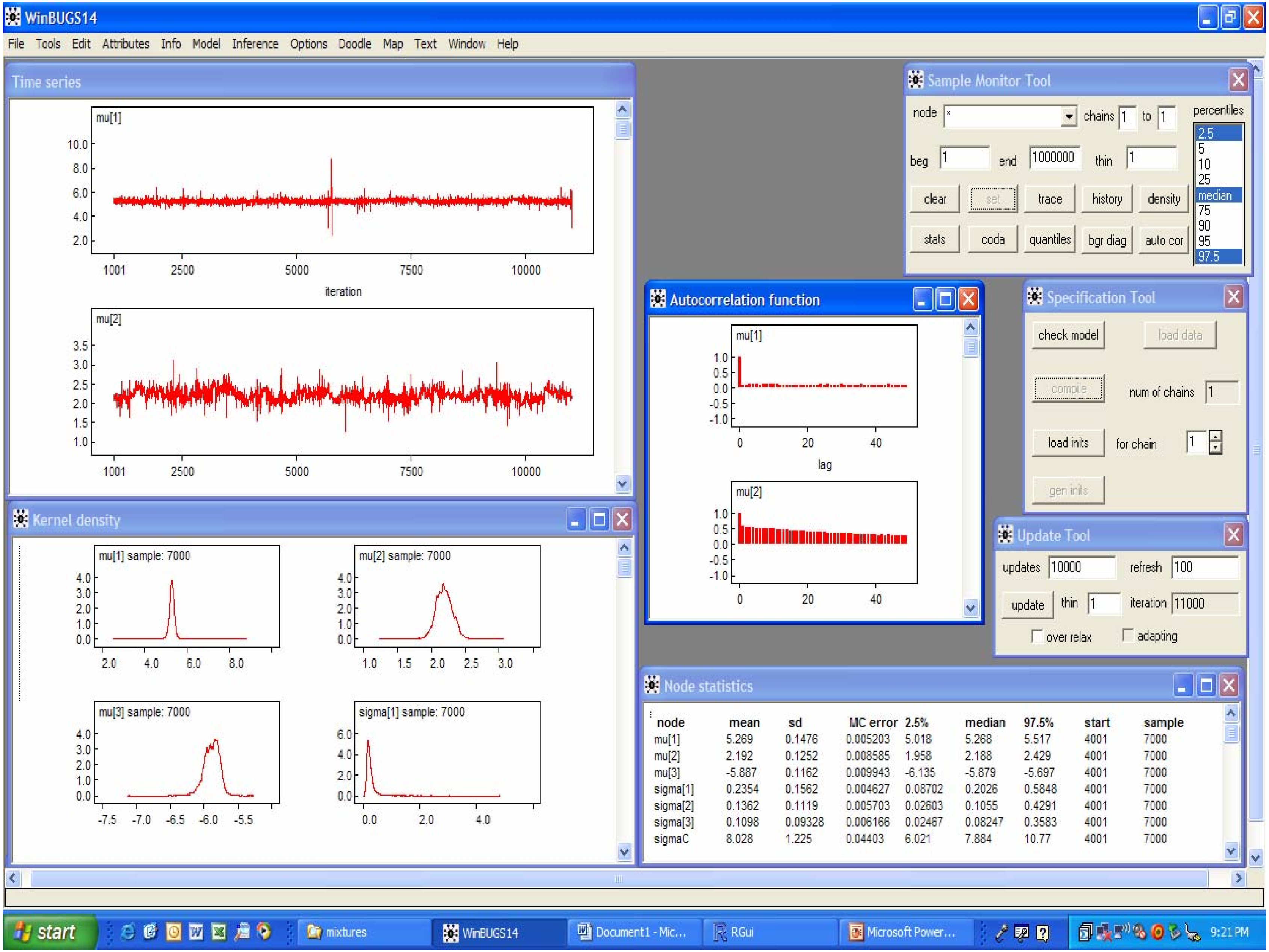
BUGS code

```
model {
  for (i in 1:K) {
      for (j in 1:n) {
         Y[i, j] ~ dnorm(eta[i, j], tauC)
         eta[i, j] <- phi[i, 1] / (1 + phi[i, 2] * exp(phi[i, 3] * x[j]))
      }
      phi[i, 1] <- exp(theta[i, 1])
      phi[i, 2] <- exp(theta[i, 2]) - 1
      phi[i, 3] <- -exp(theta[i, 3])
     for (k in 1:3) {
       theta[i, k] ~ dnorm(mu[k], tau[k])
      }
  }
  tauC ~ dgamma(1.0E-3, 1.0E-3)
  sigmaC <- 1 / sqrt(tauC)
  varC <- 1 / tauC
  for (k in 1:3) {
    mu[k] ~ dnorm(0, 1.0E-4)
    tau[k] ~ dgamma(1.0E-3, 1.0E-3)
    sigma[k] <- 1 / sqrt(tau[k])
  }
}
```

Data and initial values

```
list(n=7,K=5,x=c(118.00,484.00,664.00,1004.00,1231.00,1372.00,1582.00),
    Y=structure(.Data=c(30.00,58.00,87.00,115.00,120.00,142.0,145.00,33.
    00,69.00,111.00,156.00,172.00,203.00,203.00,30.00,51.00,75.00,108.0
    0,115.00,139.00,140.00,32.00,62.00,112.00,167.00,179.00,209.00,214.
    00,30.00,49.00, 81.00,125.00,142.00,174.00,177.00),.Dim=c(5, 7)))
```

```
list(theta=structure(.Data=c(5,2,-6,5,2,-6,5,2,-6,5, 2, -6,5,2,-6),.Dim=c(5,
    3)),mu=c(5,2,-6),tau=c(20,20,20), tauC=20)
```


WinBUGS14
File Tools Edit Attributes Info Model Inference Options Doodle Map Text Window Help
Time series
mu[1]
iteration
mu[2]
Kernel density
mu[1] sample: 7000
mu[2] sample: 7000
mu[3] sample: 7000
sigma[1] sample: 7000
Sample Monitor Tool
node
chains 1 to 1
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clear set trace history density
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Autocorrelation function
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Specification Tool
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Node statistics
node mean sd MC error 2.5% median 97.5% start sample
mu[1] 5.269 0.1476 0.005203 5.018 5.268 5.517 4001 7000
mu[2] 2.192 0.1252 0.008585 1.958 2.188 2.429 4001 7000
mu[3] -5.887 0.1162 0.009943 -6.135 -5.879 -5.697 4001 7000
sigma[1] 0.2354 0.1562 0.004627 0.08702 0.2026 0.5848 4001 7000
sigma[2] 0.1362 0.1119 0.005703 0.02603 0.1055 0.4291 4001 7000
sigma[3] 0.1098 0.09328 0.006166 0.02467 0.08247 0.3583 4001 7000
sigmaC 8.028 1.225 0.04403 6.021 7.884 10.77 4001 7000
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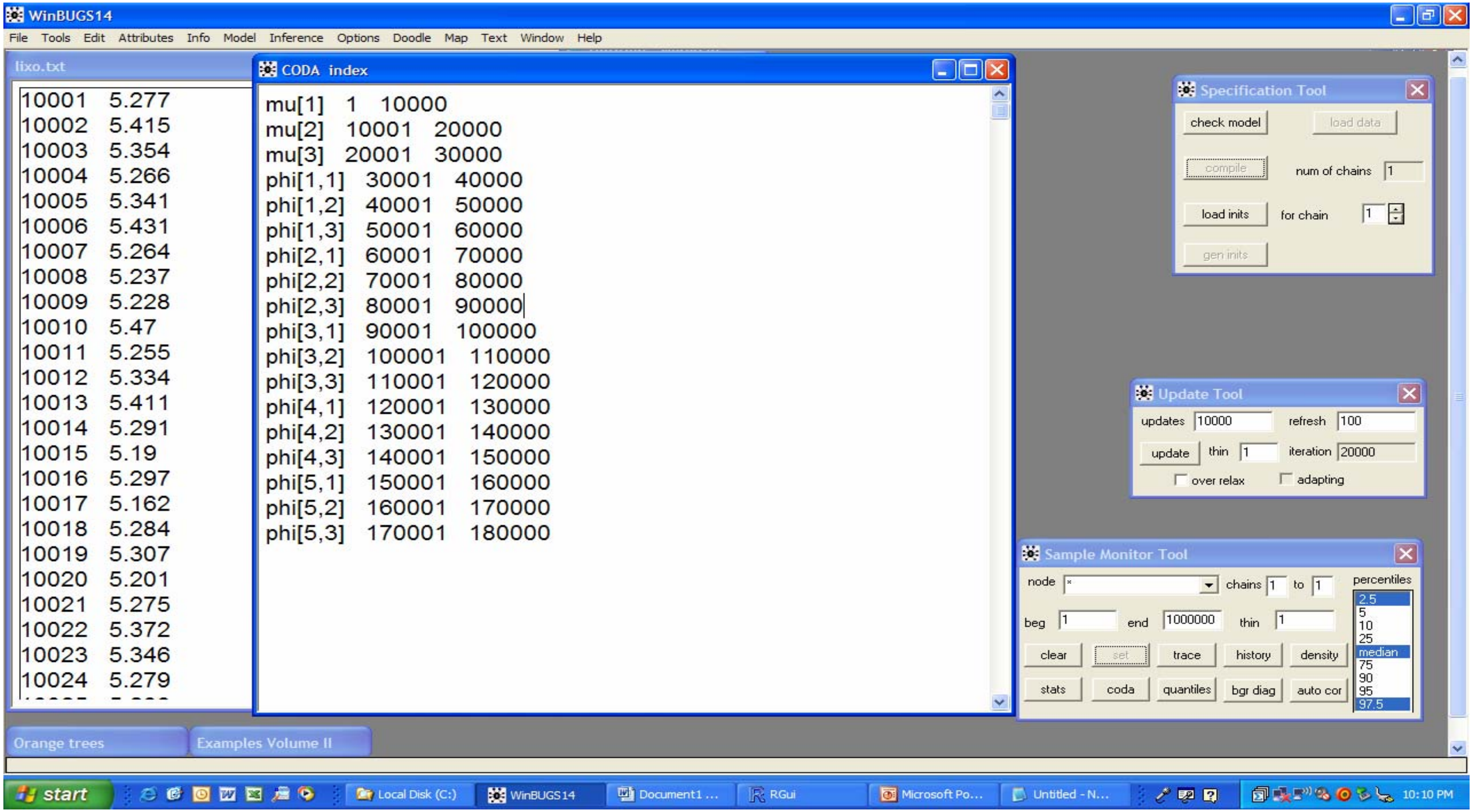
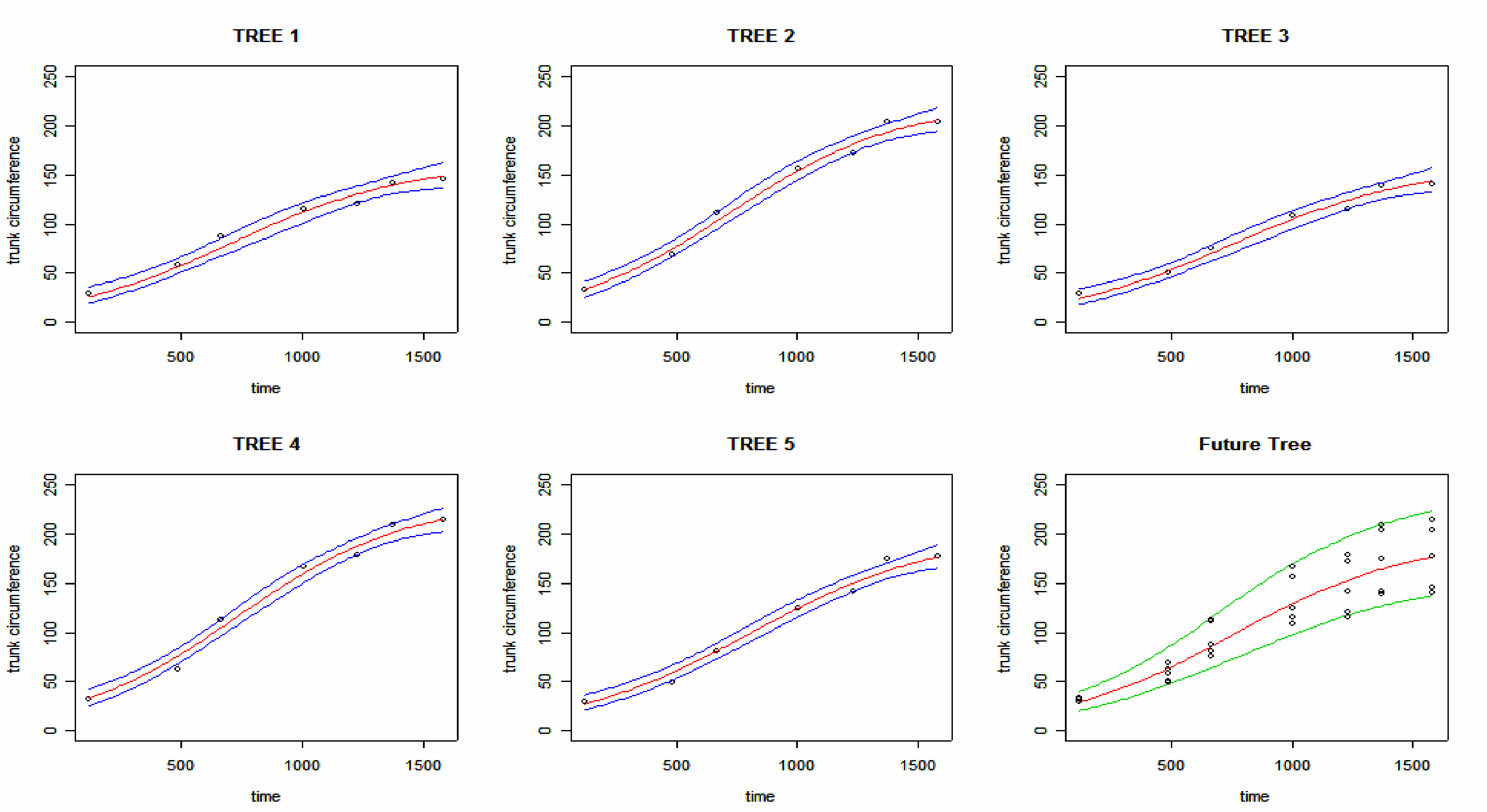
Saving MCMC sequences



Posterior densities

independent prior



Posterior densities

multivariate prior

$$\theta = (\theta_1, \theta_2, \theta_3)', \mu = (\mu_1, \mu_2, \mu_3)$$

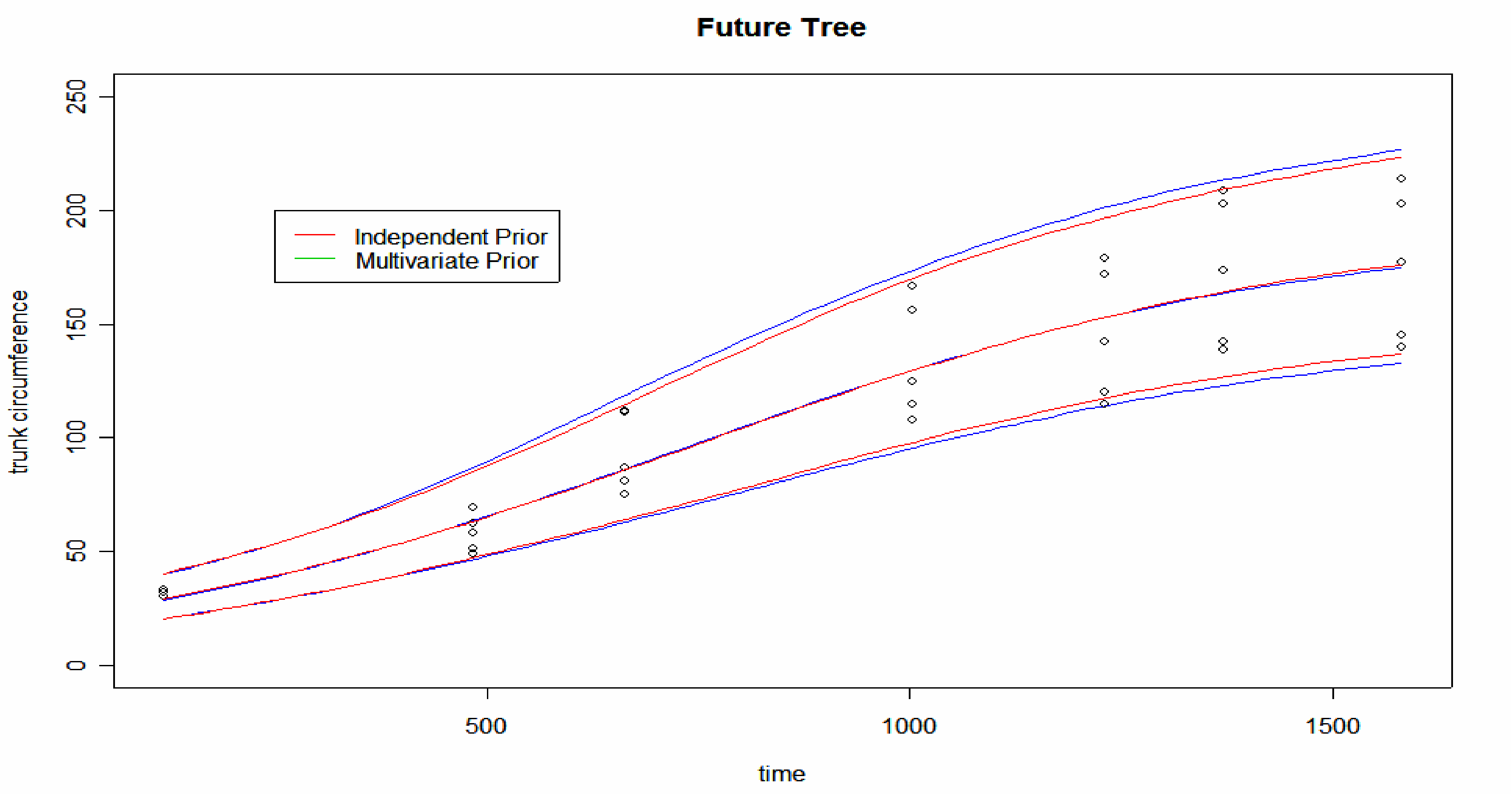
$$\theta \mid \mu, \Sigma \sim N(\mu, \Sigma)$$

$$\mu \sim N(0_3, 100000 I_3)$$

$$\Sigma^{-1} \sim Wishart(3, 0.1 I_3)$$

$$\tau_C \sim G(0.001, 0.001)$$

Posterior predictive
Independent x multivariate priors



Example 2: Multivariate hierarchical model

Goldstein et al. (1993) present an analysis of examination results from inner London schools. They use hierarchical or multilevel models to study the between-school variation, and calculate school-level residuals in an attempt to differentiate between `good' and `bad' schools.

Standardized mean examination scores (Y) were available for 1978 pupils from 38 different schools. The median number of pupils per school was 48, with a range of 1--198.

Pupil-level covariates included **gender** plus a standardized London Reading Test (**LRT**) score and a verbal reasoning (**VR**) test category (1, 2 or 3, where 1 represents the highest ability group) measured when each child was aged 11. Each school was classified by **gender intake** (all girls, all boys or mixed) and **denomination** (Church of England, Roman Catholic, State school or other); these were used as categorical school-level covariates.

LTR:London Reading Test (LRT) ;VR: verbal reasoning (VR) test

Girl: gender;G:Girls' school;B:Boys' school;CE:CE school;

RC: RC school;OS:other school.

Model and prior distribution

y_{ij}:standardized mean examination scores, for pupil i and school j.

$$y_{ij} \sim N\left(x_{ij}^T\alpha_j + (z_{ij}^T, w_j^T)\beta, \tau_{ij}^{-1}\right)$$

$$\log(\tau_{ij}^{-1}) = \theta + \phi \mathrm{LRT}_{ij}$$

$$\alpha_j = (\alpha_{1j}, \alpha_{2j}, \alpha_{3j})', \beta = (\beta_1, \beta_2, \beta_3, \delta_4, \delta_5, \delta_6, \delta_7, \delta_8)'$$

$$x_{ij}^T = (1, \mathrm{LTR}_{ij}, \mathrm{VR1}_{ij}); z_{ij}^T = (\mathrm{LTR}_{ij}^2, \mathrm{VR2}_{ij}, \mathrm{Girl}_{ij}); w_j^T = (\mathrm{G}_j, \mathrm{B}_j, \mathrm{CE}_j, \mathrm{RC}_j, \mathrm{OS}_j)$$

$$\beta_j \sim N(0,10000), \theta \sim N(0,10000), \phi \sim N(0,10000), \alpha_j \sim N(\gamma, \Omega),$$

$$\gamma \sim N(0_3, 1000 I_3), \Omega^{-1} \sim W(3, \Omega_0^{-1})$$

$$\Omega_0^{-1} = \begin{pmatrix} 0.1 & 0.005 & 0.005 \\ 0.005 & 0.1 & 0.005 \\ 0.005 & 0.005 & 0.1 \end{pmatrix}$$

Prior - WinBugs code

```
#Priors for fixed effects:
for (k in 1 : 8) {
    beta[k] ~ dnorm(0.0, 0.0001)
}
theta ~ dnorm(0.0, 0.0001)
phi ~ dnorm(0.0, 0.0001)

# Priors for random coefficients:
for (j in 1 : M) {
   alpha[j,1:3] ~ dmnorm(gamma[1:3 ], T[1:3 ,1:3 ]);
   alpha1[j] <- alpha[j,1]
}
# Hyper-priors:
gamma[1:3] ~ dmnorm(mn[1:3 ], prec[1:3 ,1:3 ]);
T[1:3 ,1:3 ] ~ dwish(R[1:3 ,1:3 ], 3)


mn = c(0, 0, 0)
prec = structure(.Data = c(0.0001, 0, 0, 0, 0.0001, 0, 0,0, 0.0001), .Dim = c(3, 3))
R = structure(.Data = c(0.1, 0.005, 0.005, 0.005, 0.1, 0.005, 0.005, 0.005, 0.1), .Dim = c(3, 3))
```

A 1000 update burn in followed by a further 10000 updates gave the parameter estimates

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
beta[1]	2.597E-4	9.65E-5	2.435E-6	7.334E-5	2.591E-4	4.497E-4	1001	10000
beta[2]	0.4099	0.06203	0.00341	0.2901	0.4093	0.53	1001	10000
beta[3]	0.17	0.04802	0.001491	0.07532	0.1698	0.2651	1001	10000
beta[4]	0.1147	0.1319	0.005749	-0.1406	0.112	0.3776	1001	10000
beta[5]	0.04825	0.1048	0.005128	-0.1645	0.04993	0.2435	1001	10000
beta[6]	-0.2871	0.1833	0.007287	-0.6438	-0.2852	0.07716	1001	10000
beta[7]	0.1477	0.1041	0.003529	-0.0575	0.1489	0.352	1001	10000
beta[8]	-0.1544	0.1885	0.007909	-0.5197	-0.1577	0.2302	1001	10000
gamma[1]	-0.6601	0.0982	0.006072	-0.8602	-0.6577	-0.4751	1001	10000
gamma[2]	0.03146	0.01012	1.33E-4	0.0113	0.03149	0.05154	1001	10000
gamma[3]	0.9447	0.08546	0.00466	0.7791	0.9433	1.111	1001	10000
phi	-0.002735	0.002834	3.034E-5	-0.0082	-0.002751	0.002699	1001	10000
theta	0.5804	0.03255	2.965E-4	0.5161	0.5807	0.6438	1001	10000

Example 3: Poisson hierarchical model

Breslow and Clayton (1993) analyse data initially provided by Thall and Vail (1990) concerning seizure counts in a randomised trial of anti-convulsant threrapy in epilepsy. The table below shows the successive **seizure counts for 59 patients**. Covariates are:

- **treatment (0,1)**
- **8-week baseline seizure counts**,
- **age** in years. The structure of this data is shown below

Patient	y1	y2	y3	y4	Trt	Base	Age
1	5	3	3	3	0	11	31
2	3	5	3	3	0	11	30
3	2	4	0	5	0	6	25
4	4	4	1	4	0	8	36
....							
8	40	20	21	12	0	52	42
9	5	6	6	5	0	12	37
....							
59	1	4	3	2	1	12	37

$$y_{jk} \sim \text{Poisson}(\mu_{jk})$$

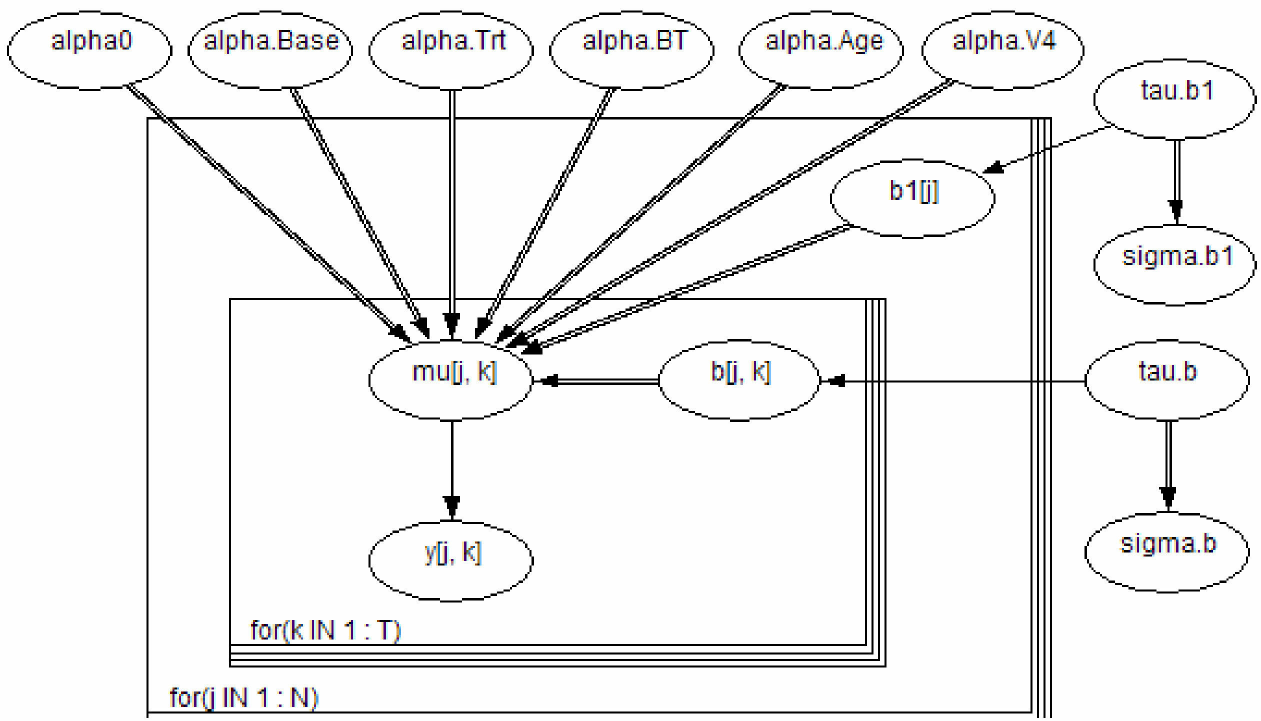
$$\log\mu_{jk} = \alpha_0 + \alpha_{Base}\log(Base_j / 4) + \alpha_{Trt}Trt_j + \alpha_{BT}Trt_j\log(Base_j / 4) + \alpha_{Age}Age_j + \alpha_{V4}V_4 + b1_j + b_{jk}$$

$$b1_j \sim \text{Normal}(0, \tau_{b1})$$

$$b_{jk} \sim \text{Normal}(0, \tau_b)$$

Coefficients and precisions are given independent "noninformative" priors.

The graphical model is below



A burn in of 5000 updates followed by a further 10000 updates gave the following parameter estimates

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
alpha.Age	0.4921	0.3721	0.0162	-0.223	0.4843	1.263	5001	10000
alpha.BT	0.3394	0.2104	0.01922	-0.04543	0.3529	0.7535	5001	10000
alpha.Base	0.8806	0.1396	0.01068	0.5825	0.883	1.153	5001	10000
alpha.Trt	-0.9319	0.4157	0.03647	-1.757	-0.949	-0.1474	5001	10000
alpha.V4	-0.1032	0.08565	0.001771	-0.2692	-0.1028	0.0647	5001	10000
alpha0	-1.421	1.254	0.05049	-3.966	-1.399	1.031	5001	10000
sigma.b	0.3642	0.04391	0.001713	0.2843	0.3622	0.4548	5001	10000
sigma.b1	0.4976	0.07162	0.002429	0.3685	0.4929	0.6504		