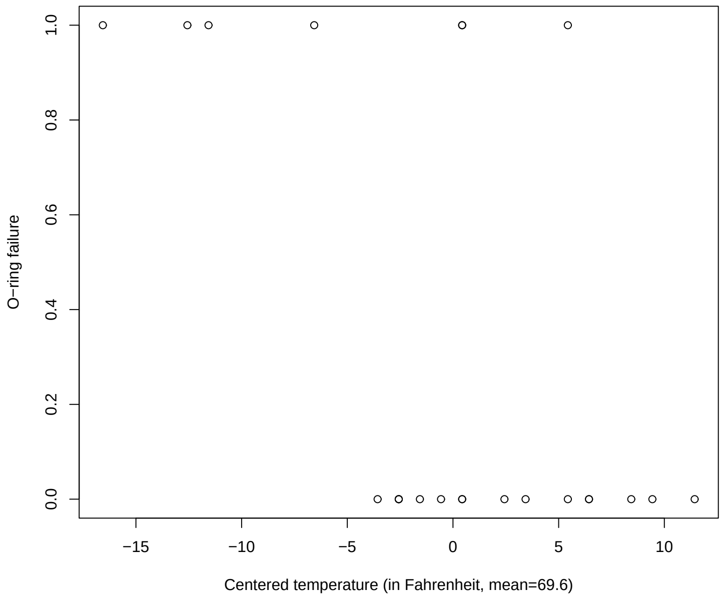
Example: O-ring failures by temperature
3 link functions and 3 prior specifications

- Christensen (1997) and Congdon (2001) analyze 23 binary observations of O-ring failures y_i (1=failure) in relation to temperature t_i (Fahrenheit).
- $y = (1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0)$
- $t = (53, 57, 58, 63, 66, 67, 67, 67, 68, 69, 70, 70, 70,$ $70, 72, 73, 75, 75, 76, 76, 78, 79, 81)$



What is $Pr(\tilde{y} = 1|\tilde{x})$, for $\tilde{x} = 31, 33, \ldots, 51$?

- Bernoulli model:

$$y_i|\theta_i \sim Bern(\theta_i)$$

  for $i = 1, \ldots, n = 23$.

- Link functions

  - Link 1: Logit

$$\log\left(\frac{\theta_i}{1-\theta_i}\right) = \alpha + \beta x_i$$

  - Link 2: Probit

$$\Phi(\theta_i) = \alpha + \beta x_i$$

  - Link 3: Complementary Log-log

$$\log(-\log(1-\theta_i)) = \alpha + \beta x_i$$

- $\alpha = -1.26$, $x_i = t_i - \bar{t}$ and $\bar{t} = 69.6$.

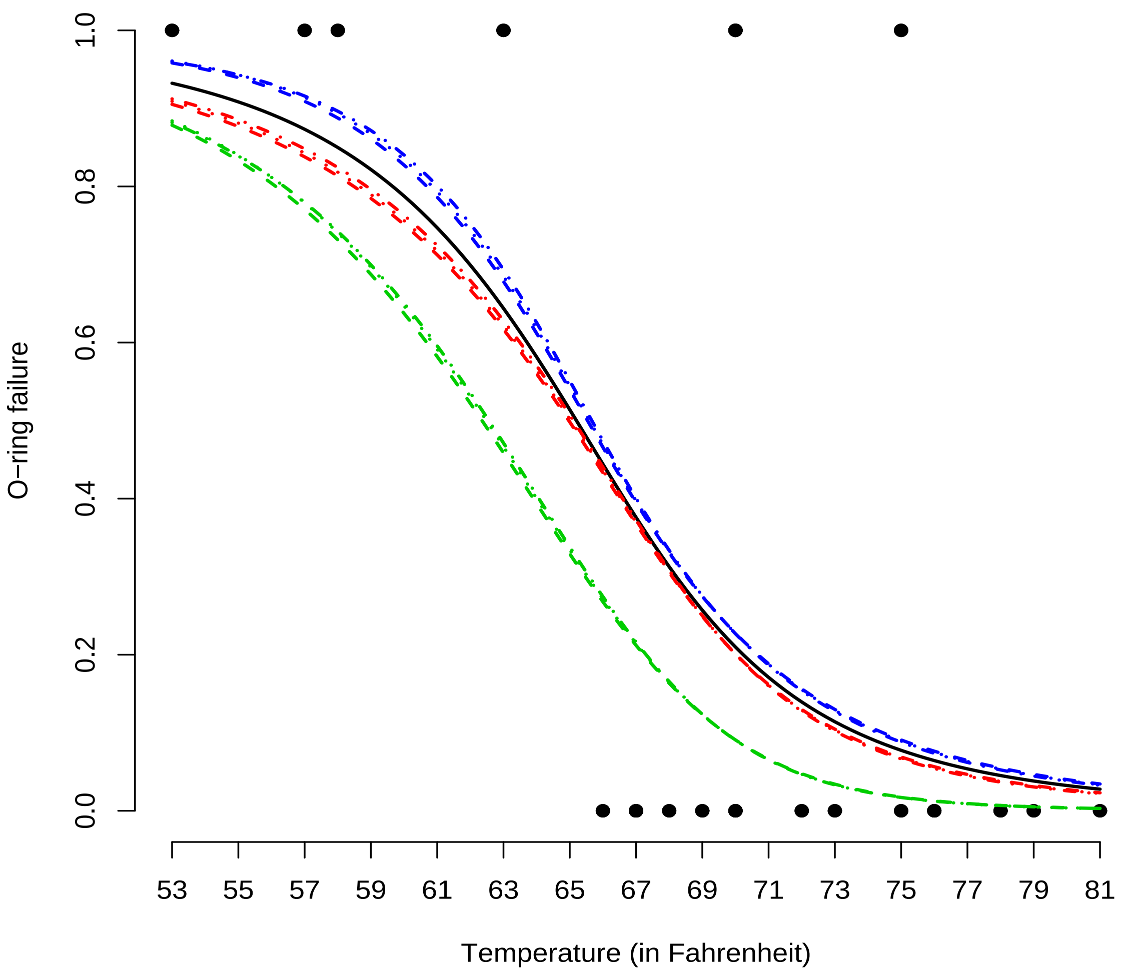
- Prior distribution - $\beta \sim N(0, V_\beta)$
  - Prior 1: $V_\beta = 1.0$
  - Prior 2: $V_\beta = 10.0$
  - Prior 3: $V_\beta = 100.0$

- Kernel of the posterior of β

$$p(\beta|y, M_j) \;\; \propto \;\; p(\beta|M_j) l_j(\theta(\beta); y)$$

  where j indexes the 9 models, corresponding the combination of 3 link functions and 3 prior variances.

Model	V_β	Link	$10^6 p(y\|M)$	$Pr(M\|y)$
M_1	1	Logit	4.00156	0.311
M_2	10	Logit	1.29728	0.101
M_3	100	Logit	0.42559	0.033
M_4	1	Probit	0.38814	0.030
M_5	10	Probit	0.12163	0.010
M_6	100	Probit	0.03754	0.003
M_7	1	Log-log	4.63591	0.361
M_8	10	Log-log	1.47942	0.115
M_9	100	Log-log	0.47139	0.037



$Pr(\tilde{y} = 1|\tilde{x})$ - Bayesian model averaging (black), logit models (red), probit models (green) and complementary log-log models (blue).