Revisiting the O-ring example

Computing $p(y|M_i)$ for $i = 1, \ldots, 9$

Model: $y_i|\theta_i \sim Bern(\theta_i)$, $i = 1, \ldots, n = 23$

Logit link: $\log\left(\frac{\theta_i}{1-\theta_i}\right) = \alpha + \beta x_i$
Probit link: $\Phi(\theta_i) = \alpha + \beta x_i$
Log-log link: $\log(-\log(1 - \theta_i)) = \alpha + \beta x_i$

Prior of $\beta \sim N(0, V_\beta)$
Prior 1: $V_\beta = 1.0$
Prior 2: $V_\beta = 10.0$
Prior 3: $V_\beta = 100.0$

$\alpha = -1.26$, $x_i = t_i - \bar{t}$ and $\bar{t} = 69.6$

Kernel of the posterior of β

$$p(\beta|y, M_j) \propto p(\beta|M_j) l_j(\theta(\beta); y)$$

where j indexes the 9 models, corresponding the combination of 3 link functions and 3 prior variances.

Marginal likelihoods

		$p(y\|M)$			
Link	V_β	True	MC	GHM	HM
Logit	1	0.966	0.968	0.967	2.099
Probit	1	0.092	0.093	0.093	0.320
Log-log	1	1.112	1.118	1.102	1.331
Logit	10	0.318	0.317	0.316	1.574
Probit	10	0.030	0.030	0.030	0.419
Log-log	10	0.361	0.362	0.363	3.477
Logit	100	0.101	0.096	0.100	1.350
Probit	100	0.009	0.009	0.010	0.404
Log-log	100	0.114	0.109	0.115	2.988

MC: Approximation based on prior draws

HM: Approximation based on posterior draws.

GHM: Approximation based on $N(-0.225, 0.0105)$.

Posterior model probabilities

Link	V_β	$Pr(M\|y)$ True	MC	GHM	HM
Logit	1	0.311	0.312	0.312	0.150
Probit	1	0.030	0.030	0.030	0.023
Log-log	1	0.358	0.361	0.356	0.095
Logit	10	0.103	0.102	0.102	0.113
Probit	10	0.010	0.010	0.010	0.030
Log-log	10	0.116	0.117	0.117	0.249
Logit	100	0.033	0.031	0.032	0.097
Probit	100	0.003	0.003	0.003	0.029
Log-log	100	0.037	0.035	0.037	0.214

Posterior model probabilities
For different link functions

		$Pr(M\|y)$			
Link	V_β	True	MC	GHM	HM
Logit	1	0.445	0.444	0.447	0.560
Probit	1	0.042	0.043	0.043	0.085
Log-log	1	0.512	0.513	0.510	0.355
Logit	10	0.449	0.447	0.446	0.288
Probit	10	0.042	0.042	0.042	0.077
Log-log	10	0.509	0.511	0.511	0.636
Logit	100	0.450	0.447	0.445	0.285
Probit	100	0.042	0.043	0.043	0.085
Log-log	100	0.509	0.510	0.511	0.630