Lecture 4:
Bayesian VAR

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Outline

VAR at a glance

Del Negro and Schorfheide (2011) says

"VARs appear to be straightforward multivariate generalizations of univariate autoregressive models. They turn out to be one of the key empirical tools in modern macroeconomics.

Sims (1980) proposed that VARs should replace large-scale macroeconometric models inherited from the 1960s, because the latter imposed incredible restrictions, which were largely inconsistent with the notion that economic agents take the effect of todays choices on tomorrows utility into account.

VARs have been used for macroeconomic forecasting and policy analysis to investigate the sources of business-cycle fluctuations and to provide a benchmark against which modern dynamic macroeconomic theories can be evaluated."

Model set up

Let $y_t = (y_{t1}, \ldots, y_{tq})'$ contain q (macroeconomic) time series observed at time t.

The (basic) VAR(p) can be written as

$$y_t = B_1 y_{t-1} + \ldots + B_p y_{t-p} + u_t$$

where

$$u_t \sim \text{i.i.d. } N(0, \Sigma)$$

and

- $B_1, \ldots, B_p$ are $(q \times q)$ autoregressive matrices
- Σ is an $(q \times q)$ variance-covariance matrix

Multivariate regression

More compactly,

$$y_t = Bx_t + u_t \qquad u_t \sim N(0, \Sigma),$$

where

- $B = (B_1, \ldots, B_p)$ is the $(q \times qp)$ autoregressive matrix,
- $x_t = (y'_{t-1}, \ldots, y'_{t-p})'$ is qp-dimensional.

Matrix normal

Let

- $Y = (y_1, \ldots, y_n)'$ is $n \times q$,
- $X = (x_1, \ldots, x_n)'$ is $n \times qp$,
- $U = (y_1, \ldots, u_n)'$ is $n \times q$.

Then

$$Y = XB' + U \qquad U \sim N(0, I_n, \Sigma).$$

Multivariate normal

Let $y_i = (y_{1i}, \ldots, y_{ni})'$ be the n-dimensional vector with the observations for time series i, for $i = 1, \ldots, q$.

Let $y = \text{vec}(Y) = (y_1', y_2', \ldots, y_q')'$.

Similarly, $\beta = \text{vec}(B')$ and $u = \text{vec}(U)$.

Then

$$y = (I_q \otimes X)\beta + u \qquad u \sim N(0, \Sigma \otimes I_n).$$

h-step ahead forecast

$$\begin{pmatrix} y_t \\ y_{t-1} \\ y_{t-2} \\ \vdots \\ y_{t-p+1} \end{pmatrix} = \begin{pmatrix} B_1 & B_2 & \cdots & B_{p-1} & B_p \\ I_q & 0 & \cdots & 0 & 0 \\ 0 & I_q & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_q & 0 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ y_{t-2} \\ y_{t-3} \\ \vdots \\ y_{t-p} \end{pmatrix} + \begin{pmatrix} u_t \\ u_{t-1} \\ u_{t-2} \\ \vdots \\ u_{t-p+1} \end{pmatrix}$$

or

$$y_t^* = A y_{t-1}^* + u_t^*.$$

Therefore, the h-step ahead forecast is

$$y_t(h) = \int F(A, y_t^*, h) p(B, \Sigma | \text{data}) d(B, \Sigma),$$

where the forecast function

$$F(A, y_t^*, h) = A^h y_t^*$$

is a highly nonlinear function of $B_1, \ldots, B_p$.

Stationarity

A VAR(p) is covariance-stationary if all values of z satisfying

$$|I_q - B_1 z - B_2 z^2 - \cdots - B_p z^p| = 0$$

lie outside the unit circle.

This is equivalent to all eigenvalues of A lying inside the unit circle.

Vector MA(∞) representation

If all eigenvalues of A lie inside the unit circle, then

$$y_t = \sum_{i=0}^{\infty} \Psi_i u_{t-i},$$

with

$$\begin{aligned}
\Psi_0 &= I_q \\
\Psi_s &= B_1\Psi_{s-1} + B_2\Psi_{s-2} + \cdots + B_p\Psi_{s-p} \quad \text{for } s = 1, 2, \ldots \\
\Psi_s &= 0 \qquad \text{for } s < 0
\end{aligned}$$

Variance Decomposition

The mean square error (MSE) of the *h*-step ahead forecast is

$$\Sigma + \Psi_1 \Sigma \Psi_1' + \cdots + \Psi_{h-1} \Sigma \Psi_{h-1}'.$$

The error $u_t \sim N(0, \Sigma)$ can be orthogonalized by

$$\varepsilon_t = A^{-1} u_t \sim N(0, D)$$

where $\Sigma = ADA'$ and D is diagonal (for instance, via singular value decomposition or Cholesky decomposition).

The MSE of the *h*-step ahead forecast can be rewritten as The conbribution of the *j*th orthogonalized innovation to the MSE is

$$d_j(a_j a_j' + \Psi_1 a_j a_j' \Psi_1' + \cdots + \Psi_{h-1} a_j a_j' \Psi_{h-1}')$$

Impulse-response function

The matrix Ψ_s has the interpretation

$$\frac{\partial y_{t+s}}{\partial u_s'} = \Psi_s,$$

that is, the (i,j) element of Ψ_s identifies the consequences of a one-unit increase in the innovation of variable j at time t for the value of variable i at time $t+s$, holding all other innovations at all dates constant.

A plot of the (i,j) element of Ψ_s,

$$\frac{\partial y_{t+s,i}}{\partial u_{s,i}},$$

as a function of s is called the *impulse-response function*.

Similar to the forecast function, the impulse-response function is also highly nonlinear on $B_1, \ldots, B_q$.

Simulated example[1]

Simulating 1000 observations from a trivariate VAR(2)

$$\mu = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \quad B_1 = \begin{pmatrix} 0.7 & 0.1 & 0.0 \\ 0.0 & 0.4 & 0.1 \\ 0.9 & 0.0 & 0.8 \end{pmatrix} \quad B_2 = \begin{pmatrix} -0.2 & 0.0 & 0.0 \\ 0.0 & 0.1 & 0.1 \\ 0.0 & 0.0 & 0.0 \end{pmatrix}$$

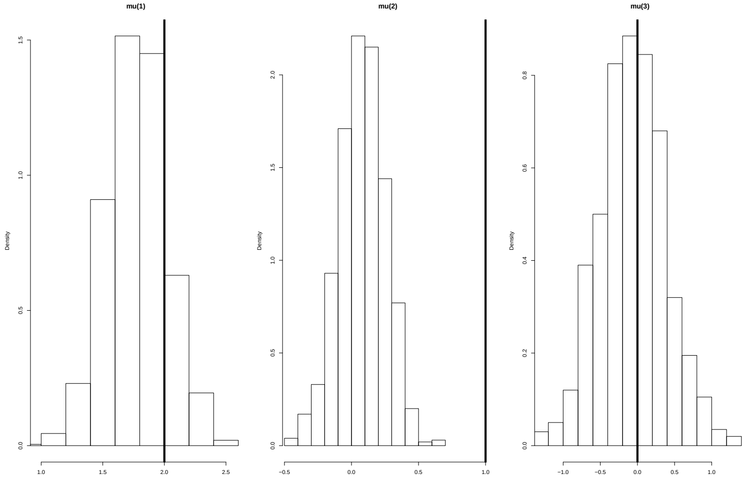
and

$$\Sigma = \begin{pmatrix} 0.26 & 0.03 & 0.00 \\ 0.03 & 0.09 & 0.00 \\ 0.00 & 0.00 & 0.81 \end{pmatrix}.$$

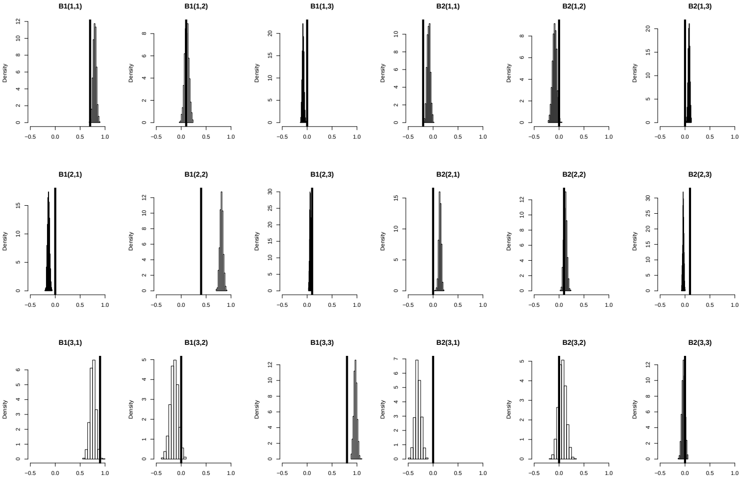
Posterior summaries based on 1000 draws.

[1] Based on Lütkepohl's (2007) Problem 2.3 and using Sims' code (later slides).

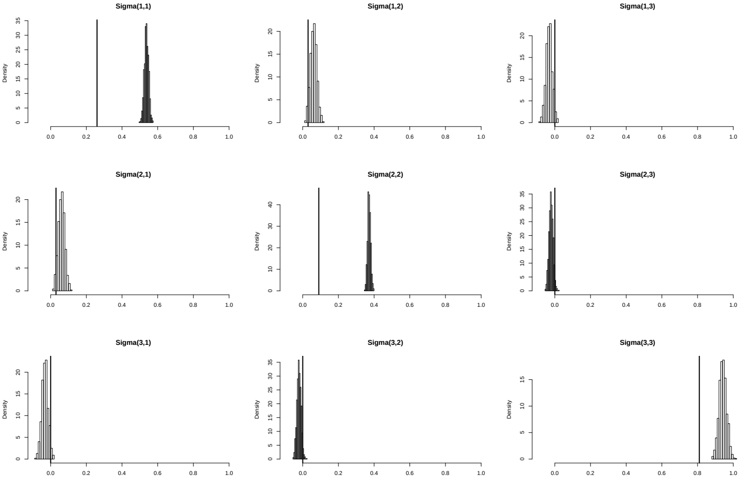
$p(\mu|\text{data})$



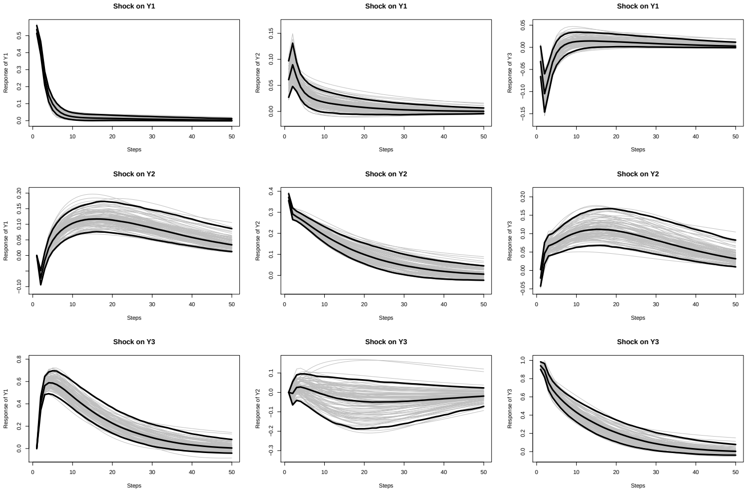
$p(B_i|\text{data})$



$p(\Sigma|\text{data})$

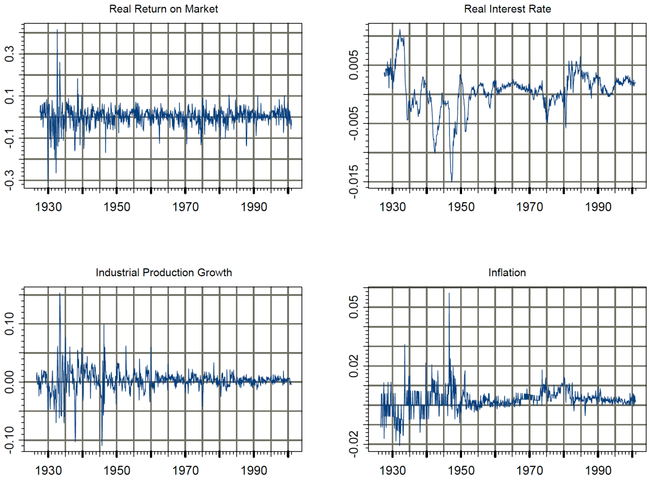


Impulse response



Real data example[2]

Monthly real stock returns, real interest rates, real industrial production growth and the inflation rate (1947.1 – 1987.12)



[2]Section 11.5 of Zivot and Wang (2003).

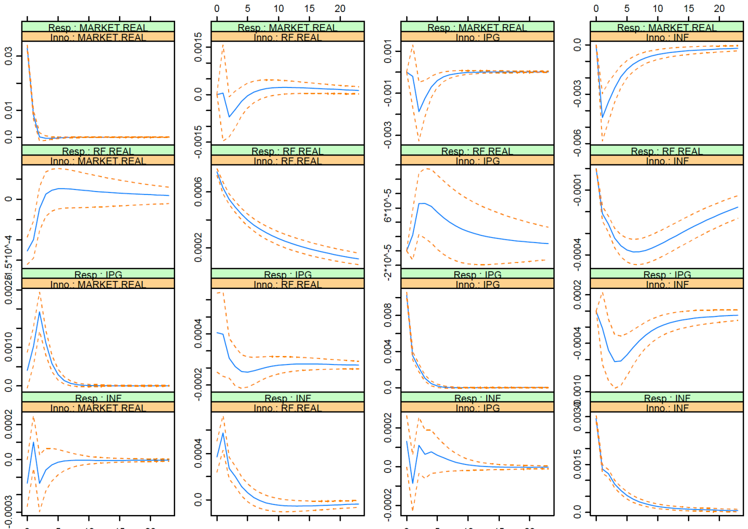
Posterior means

$$E(\mu|\text{data}) = \begin{pmatrix} 0.0074 & 0.0002 & 0.0010 & 0.0019 \end{pmatrix}'$$

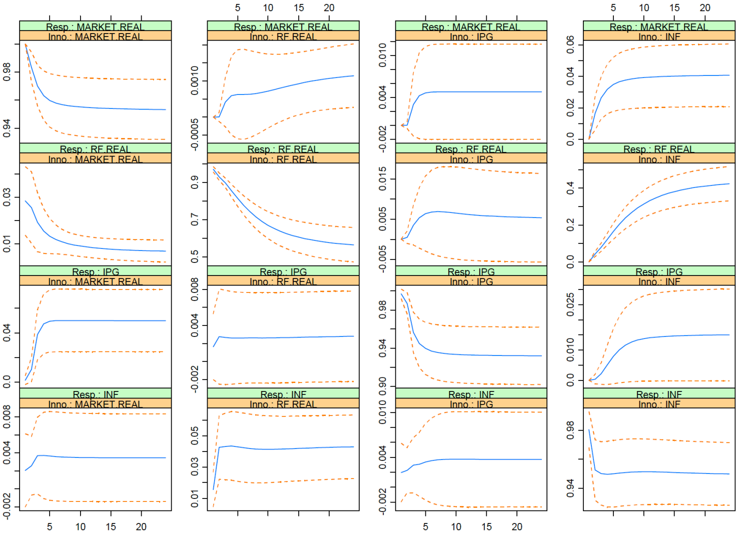
$$E(B_1|\text{data}) = \begin{pmatrix} 0.24 & 0.81 & -1.50 & 0.00 \\ 0.00 & 0.88 & -0.71 & 0.00 \\ 0.01 & 0.06 & 0.46 & -0.01 \\ 0.03 & 0.38 & -0.07 & 0.35 \end{pmatrix}$$

$$E(B_2|\text{data}) = \begin{pmatrix} -0.05 & -0.35 & -0.06 & -0.19 \\ 0.00 & 0.04 & 0.01 & 0.00 \\ -0.01 & -0.59 & 0.25 & 0.02 \\ 0.04 & -0.33 & -0.04 & 0.09 \end{pmatrix}$$

Impulse-response functions



Variance decomposition



Prior shrinkage

Recall that $\beta = vec(B')$.

Let β_i be column i of B': $B' = (\beta_1, \ldots, \beta_q)$.

Therefore,

$$\beta = (\beta_1', \ldots, \beta_q')'$$

with β_i a vector of dimension qp corresponding to all autoregressive coefficients from equation i.

Example: $q = 3$, $p = 2$

$$B = \left(\begin{array}{ccc|ccc} b_{1,11} & b_{1,12} & b_{1,13} & b_{2,11} & b_{2,12} & b_{2,13} \\ b_{1,21} & b_{1,22} & b_{1,23} & b_{2,21} & b_{2,22} & b_{2,23} \\ b_{1,31} & b_{1,32} & b_{1,33} & b_{2,31} & b_{2,32} & b_{2,33} \end{array}\right) = \begin{pmatrix} \beta_1' \\ \beta_2' \\ \beta_3' \end{pmatrix}$$

Minnesota prior

Litterman's (1980,1986) Minnesota prior advocates that

$$\beta_i \sim N(\beta_{i0}, V_{i0})$$

with V_{i0} chosen to "center" the individual equations around the random walk with drift:

$$y_{ti} = \mu_i + y_{t-1,i} + u_{ti}.$$

This amounts to:

- Shrinking the diagonal elements of B_1 toward one,
- Shrinking the remaining coefficients of $B_1, \ldots, B_p$ toward zero,

In addition:

- Shrinking the number of lags p towards one,
- Own lags should explain more of the variation than the lags of other variables in the equation.

Specifying V_{i0}

V_{i0} is diagonal with elements

$$\frac{\theta^2 \lambda^2}{k^2} \times \frac{\sigma_i^2}{\sigma_j^2}$$

for coefficients on lags of variable $j \neq i$, and

$$\frac{\lambda^2}{k^2}$$

for coefficients on own lags.

The error matrix Σ is assumed to be diagonal, fixed and known

$$\Sigma = \text{diag}(\sigma_1^2, \ldots, \sigma_q^2).$$

Other priors on (β, Σ)

Kadiyala and Karlsson (1993,1997) extend the Minnesota prior:

- Normal-Wishart prior

$$N(\beta_0, \Sigma \otimes \Omega) IW(\Sigma_0, \alpha)$$

- Jeffreys' prior

$$p(\beta, \Sigma) \propto |\Sigma|^{-(q+1)/2}$$

- Normal-Diffuse prior

$$\beta \sim N(\beta_0, V_0) \quad p(\Sigma) \propto |\Sigma|^{-(q+1)/2}$$

- Extended Natural Conjugate (ENC) prior

Kadiyala and Karlsson (1993) use MC integration.
Kadiyala and Karlsson (1997) use Gibbs sampler.

	Prior	Posterior
Minnesota	$\gamma_i \sim N(\tilde{\gamma}_i, \tilde{\Sigma}_i)$, Ψ fix and diagonal	$\gamma_i \vert \mathbf{y} \sim N(\bar{\gamma}_i, \bar{\Sigma}_i)$
Diffuse	$p(\gamma, \Psi) \propto \vert\Psi\vert^{-(m+1)/2}$	$\Gamma\vert\mathbf{y} \sim MT(\mathbf{Z}'\mathbf{Z}, (\mathbf{Y} - \mathbf{Z}\hat{\Gamma})'$ $\times(\mathbf{Y} - \mathbf{Z}\hat{\Gamma}), \hat{\Gamma}, T - k)$
Normal-Wishart	$\gamma\vert\Psi \sim N(\tilde{\gamma}, \Psi \otimes \tilde{\Omega})$, $\Psi \sim iW(\tilde{\Psi}, \alpha)$	$\Gamma \mid \mathbf{y} \sim MT(\bar{\Omega}^{-1}, \bar{\Psi}, \bar{\Gamma}, T + \alpha)$
Normal-Diffuse	$\gamma \sim N(\tilde{\gamma}, \tilde{\Sigma})$, $p(\Psi) \propto \vert\Psi\vert^{-(m+1)/2}$	$p(\gamma\vert y) \propto \exp\{-(\gamma - \tilde{\gamma})'\tilde{\Sigma}^{-1}$ $\times(\gamma - \tilde{\gamma})/2\} \times \vert(\mathbf{Y} - \mathbf{Z}\hat{\Gamma})'(\mathbf{Y} - \mathbf{Z}\hat{\Gamma})$ $+(\Gamma - \hat{\Gamma})'\mathbf{Z}'\mathbf{Z}(\Gamma - \hat{\Gamma})\vert^{-T/2}$
Extended Natural Conjugate	$p(\Delta) \propto \vert\tilde{\Psi} + (\Delta - \tilde{\Delta})'$ $\times\tilde{\mathbf{M}}(\Delta - \tilde{\Delta})\vert^{-\alpha/2}$ or independent multivariate t's for each equation, $\Psi\vert\Delta \sim iW(\tilde{\Psi} + (\Delta - \tilde{\Delta})'$ $\times\tilde{\mathbf{M}}(\Delta - \tilde{\Delta}), \alpha)$	$p(\Delta\vert\mathbf{y}) \propto \vert\bar{\Psi} + (\Delta - \bar{\Delta})'$ $\times\mathbf{M}(\Delta - \bar{\Delta})\vert^{-(T+\alpha)/2}$ $\Psi\vert\Delta, \mathbf{y} \sim iW$ $(\tilde{\Psi} + (\Delta - \bar{\Delta})'\mathbf{M}(\Delta - \bar{\Delta}), T + \alpha)$

Example

Kadiyala and Karlsson (1997) revisited Litterman (1986):

- Annual growth rates of real GNP (RGNPG),
- Annual inflation rates (INFLA),
- Unemployment rate (UNEMP)
- Logarithm of nominal money supply (M1)
- Logarithm of gross private domestic investment (INVEST),
- Interest rate on four- to six-month commercial paper (CPRATE)
- Change in business inventories (CBI).

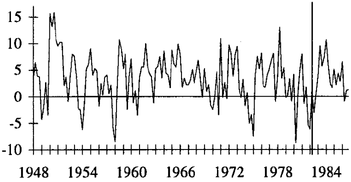
Quarterly data from 1948:1 to 1981:1 (133 observations).

Out-of-sample forecast: 1980:2 to 1986:4

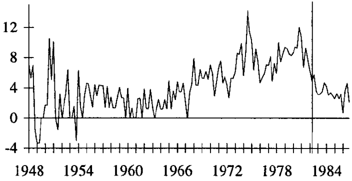
$q = 7,\ p = 6 \ \Rightarrow \ \ 7 + 6(7^2) = 301$ parameters.

Time series

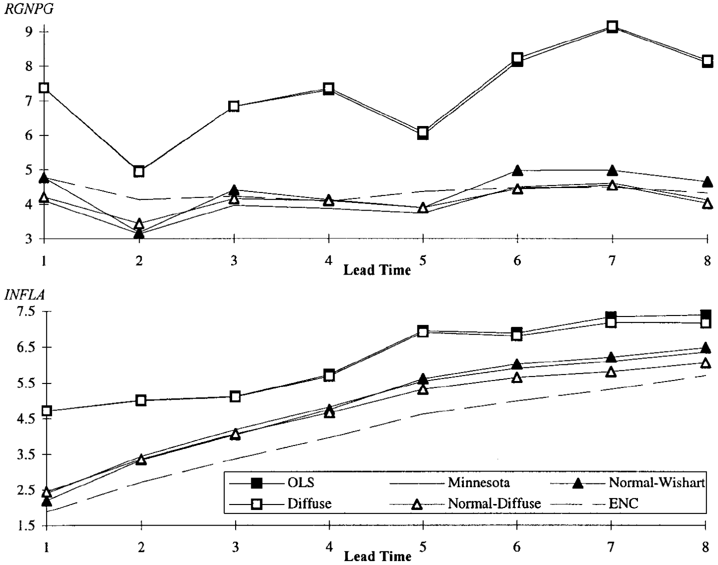
RGNPG
Annual Growth Rate in Real GNP



INFLA
Annual Inflation Rate



Root mean square error



They argue that

> *... our preferred choice is the Normal-Wishart when the prior beliefs are of the Litterman type.*

and

> *For more general prior beliefs ... the Normal-Diffuse and EN priors are strong alternatives to the Normal-Wishart.*

Structural VAR

Rubio-Ramírez, Waggoner and Zha (2010) say that

> *"Since the seminal work by Sims (1980), identification of structural vector autoregressions (SVARs) has been an unresolved theoretical issue.*
>
> *Filling this theoretical gap is of vital importance because impulse responses based on SVARs have been widely used for policy analysis and to provide stylized facts for dynamic stochastic general equilibrium (DSGE) models."*

Example

Let

- $P_{c,t}$ is the price index of commodities.
- Y_t is output.
- R_t is the nominal short-term interest rate.

Trivariate SVAR(1) representation:

$$\begin{aligned}
a_{11}\Delta \log P_{c,t} + 0.0 \log Y_t + a_{31} R_t &= c_1 + b_{11}\Delta \log P_{c,t-1} + b_{21}\Delta \log Y_{t-1} + b_{31} R_{t-1} + \varepsilon_{1,t} \\
a_{12}\Delta \log P_{c,t} + a_{22} \log Y_t + 0.0 R_t &= c_2 + b_{12}\Delta \log P_{c,t-1} + b_{22}\Delta \log Y_{t-1} + b_{32} R_{t-1} + \varepsilon_{2,t} \\
a_{13}\Delta \log P_{c,t} + a_{23} \log Y_t + a_{33} R_t &= c_3 + b_{13}\Delta \log P_{c,t-1} + b_{23}\Delta \log Y_{t-1} + b_{33} R_{t-1} + \varepsilon_{3,t}
\end{aligned}$$

1st eq. monetary policy equation.
2nd eq. characterizes behaviour of finished-goods producers.
3rd eq. commodity prices are set in active competitive markets.

Model set up

The (basic) SVAR(p) can be written as

$$A_0 y_t = A_1 y_{t-1} + \ldots + A_p y_{t-p} + u_t \qquad u_t \sim \text{i.i.d. } N(0, I_q),$$

where

- $A = (A_1, \ldots, A_p)$
- $B_i = A_0^{-1} A_i \quad i = 1, \ldots, p$
- $B = A_0^{-1} A$
- $\Sigma = (A_0 A_0')^{-1}$

Much of the SVAR literature involves exactly identified models.

Exact identification

Define g such that $g(A_0, A) = (A_0^{-1}A, (A_0 A_0')^{-1})$.

Consider an SVAR with restrictions represented by R.

Definition: The SVAR is exactly identified if and only if, for almost any reduced-form parameter point (B, Σ), there exists a unique structural parameter point $(A_0, A) \in R$ such that $g(A_0, A) = (B, \Sigma)$.

Waggoner and Zha (2003) developed an efficient MCMC algorithm to generate draws from a restricted A_0 matrix.

Illustration 1

$$A_0 = \begin{pmatrix} & PS & PS & MP & MD & Inf \\ \log Y & a_{11} & a_{12} & 0 & a_{14} & a_{15} \\ \log P & 0 & a_{22} & 0 & a_{14} & a_{25} \\ R & 0 & 0 & a_{33} & a_{34} & a_{35} \\ \log M & 0 & 0 & a_{43} & a_{44} & a_{45} \\ \log P_c & 0 & 0 & 0 & 0 & a_{15} \end{pmatrix}$$

where

- $\log Y$: log gross domestic product (GDP)
- $\log P$: log GDP deflator
- R: nominal short-term interest rate
- $\log M$: log M3
- $\log P_c$: log commodity prices

and

- MP: monetary policy (central bank's contemporaneous behavior)
- Inf: commodity (information) market
- MD: money demand equation
- PS: production sector

Illustration 2

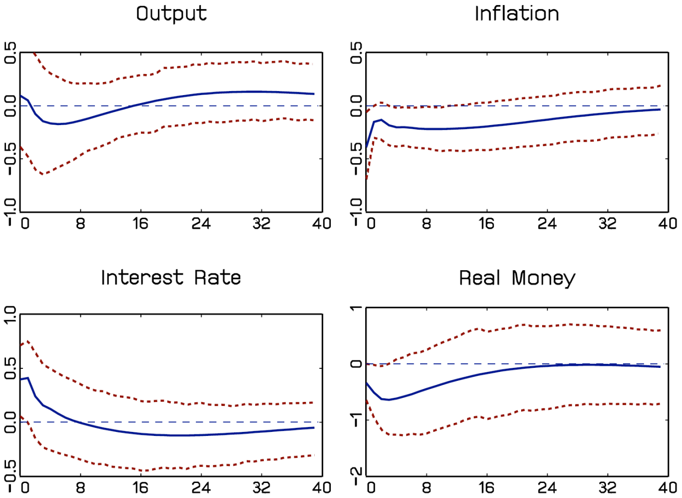
$$A_0 = \begin{pmatrix} & PCOM & M2 & R & Y & CPI & U \\ Inform & X & X & X & X & X & X \\ MP & 0 & X & X & 0 & 0 & 0 \\ MD & 0 & X & X & X & X & 0 \\ Prod & 0 & 0 & 0 & X & 0 & 0 \\ Prod & 0 & 0 & 0 & X & X & 0 \\ Prod & 0 & 0 & 0 & X & X & X \end{pmatrix}$$

where

- PCOM: Price index for industrial commodities
- M2: Real money
- R: Federal funds rate (R)
- Y: real GDP interpolated to monthly frequency
- CPI: Consumer price index (CPI)
- U: Unemployment rate (U)

- Inform: Information market
- MP: Monetary policy rule
- MD: Money demand
- Prod: Production sector of the economy

Monetary Policy Shock



VAR-GARCH

Pelloni and Polasek (2003) introduce the VAR model with GARCH errors as

$$y_t = \sum_{i=1}^{p} B_i y_{t-i} + u_t$$

where

$$u_t \sim N(0, \Sigma_t)$$

and

$$\text{vech}(\Sigma_t) = \alpha_0 + \sum_{i=1}^{r} A_i \text{vech}(\Sigma_{t-i}) + \sum_{i=1}^{s} \Theta_i \text{vech}(u_{t-i} u'_{t-i})$$

Example

German, U.S., and U.K. quarterly data sets over the period 1968-1998. Variables are logs of aggregate employment and of the employment shares of the manufacturing, finance, trade, and construction sectors for U.S. and U.K.

Table IVa. Bayes factors for model selection using posterior log-marginal likelihoods.

$\log BF_{21}$	Country		
	Germany	U.K.	U.S.
VAR	239.04	353.67	100.09
EC-VARCH	3.15	2.83	4.39
CEC-VARCH	11.79	6.71	4.79
COIN-VARCH 1	2.41	9.70	2.36
COIN-VARCH 2	6.76	1.31	10.11

VAR-SV

Uhlig (1997) introduced stochastic volatility (SV) for the error term in BVARs:

$$y_t = \sum_{i=1}^{p} B_i y_{t-i} + u_t,$$

where

$$u_t \sim N(0, \Sigma_t) \quad \text{and} \quad \Sigma_t^{-1} = L_t L_t',$$

and dynamics

$$\begin{aligned} \Sigma_{t+1}^{-1} &= \frac{L_t \Theta_t L_t'}{\lambda} \\ \Theta_t &\sim B_q\left(\frac{\nu + pq}{2}, \frac{1}{2}\right). \end{aligned}$$

TVP-VAR-SV

Primiceri (2005) discusses VARs with time varying coefficients and stochastic volatility

$$y_t = \sum_{i=1}^{p} B_{it} y_{t-i} + u_t \qquad u_t \sim N(0, \Sigma_t)$$

with

$$\Sigma_t = (A_t)^{-1} D_t (A_t')^{-1},$$

and

$$A_t = \begin{pmatrix} 1 & 0 & \cdots & 0 \\ \alpha_{21,t} & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ \alpha_{q1,t} & \cdots & \alpha_{q,q-1,t} & 1 \end{pmatrix} \quad D_t = \begin{pmatrix} \sigma^2_{1,t} & 0 & \cdots & 0 \\ 0 & \sigma^2_{2,t} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & \sigma^2_{q,t} \end{pmatrix}$$

Dynamics

VAR coefficients:

$$B_t = B_{t-1} + \nu_t \qquad \nu_t \sim N(0, Q)$$

Cholesky coefficients:

$$\alpha_t = \alpha_{t-1} + \xi_t \qquad \xi_t \sim N(0, S)$$

Stochastic volatility:

$$\log \sigma_t = \log \sigma_{t-1} + \eta_t \qquad \eta_t \sim N(0, W)$$

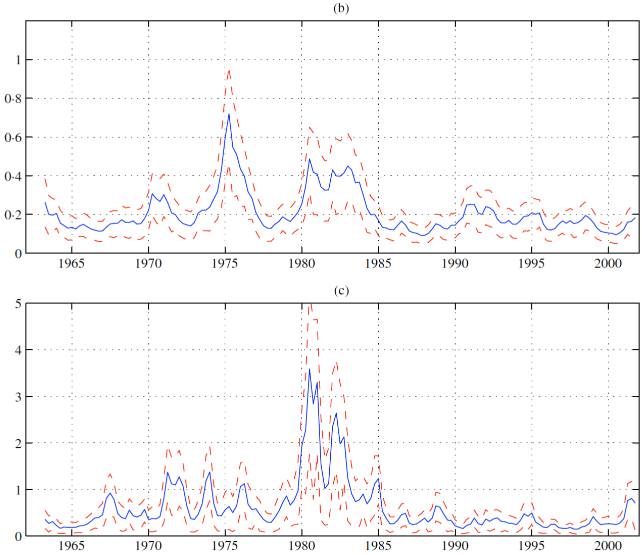


FIGURE 1

Posterior mean, 16-th and 84-th percentiles of the standard deviation of (a) residuals of the inflation equation, (b) residuals of the unemployment equation and (c) residuals of the interest rate equation or monetary policy shocks

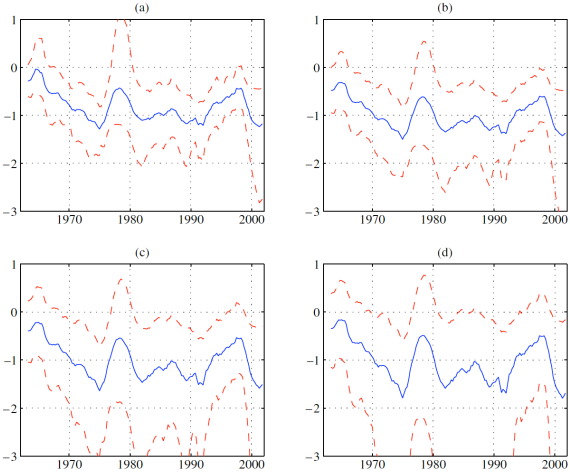


FIGURE 6

Interest rate response to a 1% permanent increase of unemployment with 16-th and 84-th percentiles. (a) Simultaneous response, (b) response after 10 quarters, (c) response after 20 quarters, (d) response after 60 quarters

See Nakajima, Kasuya and Watanabe (2011) for an application to the Japanese economy.

Curse of dimensionality

VAR(1) case[3]

Small: $q = 3 \;\Rightarrow$ 15 parameters

Medium: $q = 20 \;\Rightarrow$ 610 parameters

Large: $q = 131 \;\Rightarrow$ 25,807 parameters

[3] Small, Medium and Large are based on the VAR specifications of Bańura, Giannone and Reichlin (2010).

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