BAYESIAN ECONOMETRICS
SPRING 2013
TAKE HOME MIDTERM EXAM
DUE DATE: Thursday, May 2nd 2013, 9PM CST (by email)

Let us continue in the context of `homework 1` and `homework 2`, where we modeled the relationship between per capita spending (y) on public schools as a linear function of per capita income (x). The data is in the file `spending.txt` and

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i \qquad \epsilon_i \sim (0, \sigma^2),$$

where we entertain three models:

$\mathcal{M}_1$: $\epsilon_i \sim N(0, \sigma^2)$ and $\beta|\sigma^2 \sim N(b_0, \sigma^2 B_0)$

$\mathcal{M}_2$: $\epsilon_i \sim N(0, \sigma^2)$ and $\beta \sim N(b_0, B_0)$

$\mathcal{M}_3$: $\epsilon_i \sim t_\nu(0, \sigma^2)$ and $\beta|\sigma^2 \sim N(b_0, \sigma^2 B_0)$

with $\sigma^2 \sim IG(\eta_0/2, \eta_0 s_0^2/2)$ for $\mathcal{M}_i, i = 1, 2, 3$, and $\nu = 4.46$ for $\mathcal{M}_3$. The hyperparameters b_0, B_0, a and b are obtained such that, *a priori*, $E(\beta|\mathcal{M}_i) = (-70, 600)'$, $V(\beta|\mathcal{M}_i) = 10000 I_2$, $E(\sigma^2|\mathcal{M}_i) = 3750$ and $V(\sigma^2|\mathcal{M}_i) = 1562500$, for $i = 1, 2, 3$.

a) Compute Bayes factors B_{12}, B_{13} and B_{23}.

b) Draw $p(y_{new}|x_{new} = 800, x, y, \mathcal{M}_i)$ for $i = 1, 2, 3$.

c) Draw $p(y_{new}|x_{new} = 800, x, y)$.

d) Compute the DIC($\mathcal{M}_i$), for $i = 1, 2, 3$.

Note:

1) Recall that $B_{ij} = p(y|x, \mathcal{M}_i)/p(y|x, \mathcal{M}_j)$.

2) In a) $p(y|x, \mathcal{M}_1)$ has to be derived analytically.

3) In b) $p(y_{new}|x_{new} = 800, x, y, \mathcal{M}_1)$ has to be derived analytically.