Let us continue in the context of homework 1 and homework 2, where we modeled the relationship between per capita spending ($y$) on public schools as a linear function of per capita income ($x$). The data is in the file spending.txt and

$$y_i = \beta_0 + \beta_1 x_i + \epsilon_i \quad \epsilon_i \sim (0, \sigma^2),$$

where we entertain three models:

- $M_1$: $\epsilon_i \sim N(0, \sigma^2)$ and $\beta \sim N(b_0, \sigma^2 B_0)$
- $M_2$: $\epsilon_i \sim N(0, \sigma^2)$ and $\beta \sim N(b_0, B_0)$
- $M_3$: $\epsilon_i \sim t_\nu(0, \sigma^2)$ and $\beta \sim N(b_0, \sigma^2 B_0)$

with $\sigma^2 \sim IG(\eta_0/2, \eta_0 \nu_0^2/2)$ for $M_i, i = 1, 2, 3$, and $\nu = 4.46$ for $M_3$. The hyperparameters $b_0$, $B_0$, $a$ and $b$ are obtained such that, a priori, $E(\beta|M_i) = (-70, 600)'$, $V(\beta|M_i) = 10000I_2$, $E(\sigma^2|M_i) = 3750$ and $V(\sigma^2|M_i) = 1562500$, for $i = 1, 2, 3$.

a) Compute Bayes factors $B_{12}$, $B_{13}$ and $B_{23}$.

b) Draw $p(y_{\text{new}}|x_{\text{new}} = 800, x, y, M_i)$ for $i = 1, 2, 3$.

c) Draw $p(y_{\text{new}}|x_{\text{new}} = 800, x, y)$.

d) Compute the DIC($M_i$), for $i = 1, 2, 3$.

**Note:**

1) Recall that $B_{ij} = p(y|x, M_i)/p(y|x, M_j)$.

2) In a) $p(y|x, M_1)$ has to be derived analytically.

3) In b) $p(y_{\text{new}}|x_{\text{new}} = 800, x, y, M_1)$ has to be derived analytically.