BAYESIAN ECONOMETRICS
SPRING 2013
HOMEWORK 4
DUE DATE: June 4th 2013 (at the beginning of the class)

In class we presented the SV-AR(1) model

$$
\begin{aligned}
y_t &= \sigma_t \epsilon_t && \epsilon_t \sim N(0,1) \\
\log \sigma_t^2 &= \alpha + \beta \log \sigma_{t-1}^2 + \tau \omega_t && \omega_t \sim N(0,1),
\end{aligned}
$$

as a simple model for log-returns of financial time-series, where $h_t = \log \sigma_t^2$ is (latent) the log-volatility at time t. Let us assume that the prior for (α, β, τ^2) is such that $p(\alpha, \beta|\sigma^2) \sim N(b_0, \tau^2 B_0)$ and $\tau^2 \sim IG(\eta_0/2, \eta_0 \tau_0^2/2)$ and $h_0 \sim N(m_0, C_0)$.

Let us study the modeling of S&P500 log-returns (derived from the S&P500 index) for the following four-year periods: a) 01/01/2001-12/31/2004; b) 01/01/2005-12/31/2008; c) 01/01/2009-12/31/2012; and d) 01/01/2001-12/31/2012. That is, roughly 1000 business days for each sub-sample and 3000 overall. You can obtain the data freely from the Yahoo Finance page:

`http://finance.yahoo.com/q/hp?s=^GSPC+Historical+Prices`

Obtain posterior inference for the static parameters (α, β, τ^2) as well as the latent log-volatilities $h_1, \ldots, h_n$ for datasets a), b), c) and d), using the following three schemes:

I. Random-walk MH for $p(h_t|h_{-t}, \alpha, \beta, \tau^2, y^n)$, for $t = 1, \ldots, n$.

II. Independent MH for $p(h_t|h_{-t}, \alpha, \beta, \tau^2, y^n)$, for $t = 1, \ldots, n$, with the proposal obtained via Taylor expansion of the likelihood (see class notes).

III. The normal approximation to $\log \chi_1^2$, which leads to a Gibbs sampler

Notice that I and II will most certainly require very long chains, while III will produce bad results regarding h_t's and (α, β, τ^2). It is your call to discuss the selection of the prior hyperparameters m_0, C_0, b_0, B_0, ν_0 and τ_0^2, as well as the MCMC specifications (initial values, burn-in length, chain size and thinning). The three sub samples are intended to see how α, β and τ^2 might vary over time as well.

Bonus (20%): Use scheme I when $\epsilon_t \sim t_\nu(0,1)$ for the three datasets. Summarize posterior inference for $h_1, \ldots, h_n$ and (α, β, τ^2) for a few values of τ^2, such as $\{5, 10, 15\}$.