

BAYESIAN ECONOMETRICS

SPRING 2013

HOMEWORK 3

DUE DATE: May 21st 2013 (at the beginning of the class)

In class today (May 14th 2013) we studied the AR(1) plus noise model:

$$\begin{aligned}y_t &= x_t + \epsilon_t & \epsilon_t &\sim N(0, \sigma^2) \\x_t &= \alpha + \beta x_{t-1} + \omega_t & \omega_t &\sim N(0, \tau^2),\end{aligned}$$

for  $t = 1, \dots, n$  and  $\alpha$  and  $\beta$  known. The prior for  $\sigma^2$  and  $\tau^2$  are, respectively,  $IG(\nu_0/2, \nu_0\sigma_0^2/2)$  and  $IG(\eta_0/2, \eta_0\tau_0^2/2)$ . We devised, also in class, a (simple) MCMC scheme that iteratively samples from the following full conditionals

- $p(\sigma^2|x^n, \tau^2, y^n) \equiv p(\sigma^2|x^n, y^n)$
- $p(\tau^2|x^n, \sigma^2, y^n) \equiv p(\tau^2|x^n)$
- $p(x_0|x^n, \sigma^2, \tau^2, y^n) \equiv p(x_0|x_1, \tau^2)$
- $p(x_t|x_{-t}, y^n, \sigma^2, \tau^2) \equiv p(x_t|x_{t-1}, x_{t+1}, y_t, \tau^2)$ , for  $t = 1, \dots, n-1$ , and
- $p(x_n|x_{n-1}, y^n, \sigma^2, \tau^2) \equiv p(x_n|x_{n-1}, y_n, \tau^2)$ ,

in order to approximately sample from the joint  $p(x^n, x_0, \sigma^2, \tau^2|y^n)$ . Complete the MCMC by adding the steps to sample from  $\alpha$  and  $\beta$  when their priors are flat, i.e.  $p(\alpha) \propto 1$  and  $p(\beta) \propto 1$ .

**Real data application.** We consider the intradaily realized volatilities of Alcoa stock from 2 January 2003 to 7 May 2004 for 340 observations. The daily realized volatilities used are the sums of squares of intraday 5 min, 10 min and 20 min log returns measured in percentages; see Tsay (2005, Ch. 11). The realized volatilities are in the file `rv-alcoa.txt`. Let us use the logarithms of the daily realized volatilities as the observations in the above AR(1) plus noise model. In this case  $x_t$  is the hidden log daily volatility which is only observed through the (noisy) log realized volatilities. Use the above MCMC scheme to learn about  $x_1, \dots, x_n$  as well as  $(\alpha, \beta, \tau^2, \sigma^2)$ , in the light of the data  $y_1, \dots, y_n$ , for each one of the three series (5 min, 10 min and 20 min realized volatilities).