BAYESIAN ECONOMETRICS
SPRING 2013
HOMEWORK 2
DUE DATE: May 7th 2013 (at the beginning of the class)

Let us continue in the context of `homework 1`, where we modeled the relationship between per capita spending (y) on public schools as a linear function of per capita income (x). The data is in the file `spending.txt` and the model is

$$y_i = \alpha + \beta x_i + \epsilon_i \qquad \epsilon_i \sim t_\nu(0, \sigma^2).$$

Let us assume that only $\nu = 4.46$ is given and that $\theta = (\alpha, \beta)$ and σ^2 are unknown with the following prior specification

$$\theta \sim N(b_0, B_0) \quad \text{and} \quad \sigma^2 \sim IG(a, b)$$

where $b_0 = (-70, 600)'$, $B_0 = 10000 I_2$ and $(a, b) = (5, 15000)$, indicating weak prior information.

a) Design and implement a *random walk M-H* algorithm that samples iteratively from
$$p(\theta|\sigma^2, y, x) \quad \text{and} \quad p(\sigma^2|\theta, y, x),$$
in order to draw from $p(\theta, \sigma^2|y, x)$.

b) Rewriting $\epsilon_i \sim t_\nu(0, \sigma^2)$ as $\epsilon_i|\lambda_i \sim N(0, \lambda_i\sigma^2)$ and $\lambda_i \sim IG(\nu/2, \nu/2)$, design and implement a *Gibbs sampler* that samples iteratively from
$$p(\theta|\sigma^2, \lambda, y, x), \quad p(\sigma^2|\theta, \lambda, y, x) \quad \text{and} \quad p(\lambda_i|\theta, \sigma^2, x, y)\ (i = 1, \ldots, n),$$
in order to draw from the posterior $p(\theta, \sigma^2|y, x)$.

c) Compare the algorithms in terms of computational time (which includes convergence issues) and Monte Carlo efficiency (via effective sample size) when computing $E(\alpha|x, y)$, $E(\beta|x, y)$, $E(\sigma^2|x, y)$ and $E(y_{new}|x_{new} = 8000, x, y)$. Report and discuss your findings.

d) Generalize the above Gibbs sampler to learn ν when its prior distribution, which we will name $p_1(\nu)$, is a discrete uniform on $\{1, \ldots, 100\}$. Read Fonseca *et al.* (2008) Objective Bayesian analysis for the Student-t regression model, *Biometrika*, vol. 95, pages 325-333, discretize their noninformative prior for ν, which we will name $p_2(\nu)$, and compare both posteriors for ν, i.e. $p_1(\nu|x, y)$ and $p_2(\nu|x, y)$. Report and discuss your findings.