BAYESIAN ECONOMETRICS
SPRING 2013
HOMEWORK 1
DUE DATE: April 23rd 2013 (at the beginning of the class)

The file `spending.txt` contains data on per capita spending (y) on public schools and per capita income (x) by state in 1979 in the United States. The data are given in Greene (1997, Table 12.1, p. 541) and has been analyzed by Cribari-Neto, Ferrari and Cordeiro (2000) and Fonseca, Ferreira and Migon (2008), amongst others. Fonseca *et al.* (2008), for instance, considers the linear regression model in which an n-vector of observations $y = (y_1, \ldots, y_n)'$ satisfies

$$y = \alpha + \beta x + \varepsilon$$

where $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)'$ is the error vector and $\varepsilon_1, \ldots, \varepsilon_n$ are independent and identically distributed according to the Student-t distribution with location zero, scale parameter σ and ν degrees of freedom, commonly denoted by $t_\nu(0, \sigma)$. The n-dimensional vector $x = (x_1, \ldots, x_n)'$ contains the explanatory variable *income*. In this exercise, we will assume (unrealistically) that $\nu = 4.46$ and $\sigma = 46.7$ are known and will focus our attention on implementing a SIR algorithm to obtain, among other things, summaries of $p(\alpha, \beta|x, y)$. In addition, x (per capita income) will be scaled by 10^{-4}.

a) Implement a SIR algorithm to draw $\{(\alpha^{(1)}, \beta^{(1)}), \ldots, (\alpha^{(M)}, \beta^{(M)})\}$ from the target distribution, i.e. the posterior distribution $p(\alpha, \beta|y, x)$. The number of draws M is part of your investigation.

b) Obtain the Monte Carlo approximation to the posterior predictive $p(y_{new}|x_{new}, x, y)$ for a few values, say 5, of x_{new}. In particular, compute the Monte Carlo approximation to $E(y_{new}|x_{new}, x, y)$.

c) A Monte Carlo scheme to sample $\{y_{new}^{(1)}, \ldots, y_{new}^{(M)}\}$ from $p(y_{new}|x_{new}, x, y)$ is to sample $y_{new}^{(i)}$ from $p(y_{new}|\alpha^{(i)}, \beta^{(i)}, x_{new}, x, y) = p(y_{new}|\alpha^{(i)}, \beta^{(i)}, x, y)$, which is a $t_\nu(\alpha^{(i)}+\beta^{(i)}x_{new}, \sigma)$, for $i = 1, \ldots, M$. Compare the histogram of $\{y_{new}^{(1)}, \ldots, y_{new}^{(M)}\}$ with the Monte Carlo approximation to the posterior predictive $p(y_{new}|x_{new}, x, y)$ from b).

d) Show (empirically) that the approximation to $E(y_{new}|x_{new}, x, y)$ from b) has smaller Monte Carlo error than $\sum_{i=1}^{M} y_{new}^{(i)}/M$ from c).